КРИПТОНИТ

Format Preserving Encryption

A survey

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Statement of the problem

What is FPE?

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where:

$$\begin{aligned} t \in \mathbf{Twk} &\quad \text{is a tweak (a block cipher parameter),} \\ k \in \mathbf{Keys} &\quad \text{is a key,} \\ m \in \mathbf{Dom} &\quad \text{is a message.} \end{aligned}$$

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Tweak space might be empty: $\mathbf{Twk} = \varnothing$ (more on that later).

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Even if $\mathbf{Dom} \subseteq \{0,1\}^n$, the result $m \to E_k(m) \notin \mathbf{Dom}$ with high probability (due to its relatively small size in the real-world situations and applications).

Message space from the real world

Example (SNILS)

If we encrypt 9-digit individual insurance account number (SNILS), the result must be 9-digit ciphertext, i.e.

$$\mathtt{Dom} = \{0, 1, \ldots, 9\}^9$$

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Example (CCN)
CCN consist of the following numbers:

- **6 digits** — bank number,
- **6 digits** — account number,
- **3 digits** — checksum,

and all digits, except for account number (i.e., 9 out of 15), are publicly available. In this case:

$$\mathbf{Dom} = \{0, \dots 9\}^6.$$

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The main goal of the (non-secret) tweak is to expand the set of possible permutations;

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- The algorithm must be efficient.
- It is desirable to use existing well-studied primitives and principles: block ciphers, Feistel networks.

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Algorithm 8 Experiment Right

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function INIT
    k ←$ Keys
function O(t, m)
    return E_k^t(m)
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Formalization

Let $Adv_E^{TPRP}(\mathcal{A})$ be the advantage of the adversary $\mathcal{A}$ in the distinguishing attack, i.e.:

$$Adv_E^{TPRP}(\mathcal{A}) = \mathbb{P}[Right(\mathcal{A}) \to 1] - \mathbb{P}[Left(\mathcal{A}) \to 1].$$

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The probability $\mathbb{P}[\cdot]$:

1. is taken over random choise of permutations $\pi^t \leftarrow^R Perm(\mathtt{Dom})$ for different t and random coins of $\mathcal{A}$ (if any) — in case of Left experiment;
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The algorithm is «good» if the maximal advantage is «small».

Proposed solutions

Overview of solutions

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- General techniques: cycle walking;
- Small domain solutions: prefix encryption, various shuffling techniques;
- Primitive layer solutions (SPF);

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- One round is of the form:

$$(A, B) \to (B, A \boxplus Q),$$

where $Q = PRF_k(B, t, i, params)$, i is the round number, $params$ is some (non-secret) information, PRF is a pseudorandom function

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- Suggestion: 10 rounds of Feistel network are enough for FF1 security and 8 rounds for FF3.

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3. But: no provable security;
4. But: bad tweak mixing in FF3 $\Rightarrow$ some specific attacks on FF3;

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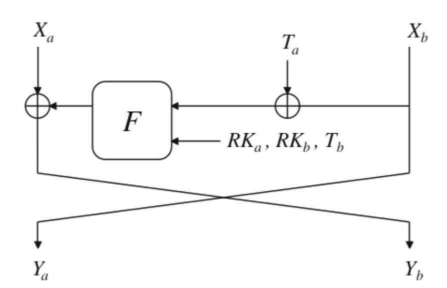
Main flaw: the key length $|sk| = 128$ is too short to guarantee the strong security bound.

FEA-2

One round of the proposed scheme:



$X_a \,||\, X_b$ — left and right blocks of the message;
$T_a \,||\, T_b$ — left and right blocks of the tweak;
$RK_a \,||\, RK_b$ — left and right blocks of the round key;

It is assumed that $|T_a| = |X_b|, |T_a| + |T_b| = 128 = |RK_a| = |RK_b|$).

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5. Encryption on the domain $\mathbf{Dom} = \{1, \ldots, N\}$ is done via embedding $\mathbf{Dom} \subseteq \{0, 1\}^n$ combined with the cycle walking idea (more on that later).
6. Linear and differential analysis, related-key attacks and threats, specific to Feistel networks over small domains, were investigated. It was claimed that for the domains of size greater than 2^8 the proposed attacks require at least 2^{64} encryptions on different parameters $t \in \mathbf{Twk}$.

Cycle walking

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- The expected number of encryption operations (before one obtains $c \in \mathbf{Dom}$) is determined by the quantity $\frac{2^n}{|\mathbf{Dom}|}$;
- Side-channel attacks (time) does not give an information to the adversary;

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The method is provably secure but requires $O(N)$ encryption operations at the initial step and $O(N)$ memory to store the table.

Attacks on the solutions

Attack on FF2

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If $c'_j = c_i$, then (with high probability) $sk_j = E_k(t_j)$, i.e., he can recover the derived key for the given tweak t_j without knowing the value of master-key k.

If the number of ciphertexts $n = 2^u$, then the collision is expected to occur after 2^{128-u} steps.

Attacks on FF1, FF3

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1. Exploit the wrong design of tweak mixing (specific for FF3). In these attacks, the adversary adaptively chooses plaintexts to be encrypted on two selected $t_1, t_2 \in \texttt{Twk}$.

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1. Exploit the wrong design of tweak mixing (specific for FF3). In these attacks, the adversary adaptively chooses plaintexts to be encrypted on two selected $t_1, t_2 \in \mathbf{Twk}$.
2. Intrinsic feature of Feistel network over small domains: the proposed number of rounds is not enough to hide the plaintext statistics (a slight bias after one round of Feistel network, which can be amplified using different tweaks $t \in \mathbf{Twk}$ for the same message).

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- The number of required texts formally exceeds the domain size;
- In fact only a minimal (even constant) number of texts are required for each $t \in \texttt{Twk}$.
- This fact was not reflected in the original security model. All the proofs were obtained in a weaker model, in which the adversary cannot make the number of requests to the oracle that exceeds the domain size.

Notation

The following notation is used:

- n — bitsize of one half of the message $m \in \mathbf{Dom}$, i.e. $\mathbf{Dom} = \{0, 1\}^{2n}$;
- N — number of different halves of the message, i.e. $N = 2^n$;
- r — number of rounds in Feistel network;
- q — number of oracle queries;
- t — time complexity (in parrots);

Short summary of attacks: 1

Year: 2004

Threat: distinguisher, **generic Feistel network**

Resources: $q_t = N^{r-2}$ encryptions queries on different $t \in \texttt{Twk}$, two messages per tweak ($q_e = 2$), time complexity $t \approx q_t q_e$

Comments: attack distinguishes Feistel network output from a random string

Short summary of attacks: 2

Year: 2016

Threat: message recovery, **generic Feistel network**

Resources: $q_t = \mathcal{O}(n \cdot N^{r-2})$ encryptions queries on different $t \in$ **Twk**, 3 messages per tweak ($q_e = 3$), time complexity $t \approx q_t$

Comments:

1. The adversary knows ciphertexts of three different messages (x, x', x^*) under tweaks $t_1, \dots, t_q$, and recovers the message x.
2. The message x' is fully known to the adversary but unrelated to x.
3. x^* and x share a common right side; only the left side of x^* is known to the adversary.
4. The attack is not adaptive; only the knowledge of plaintexts is required.

Short summary of attacks: 3

Year: 2017

Threat: Entire codebook recovery for t_1, t_2 for **FF3**.

Resources: $q_e = \mathcal{O}(N^{\frac{11}{6}})$ encryption queries on two tweaks $t_1, t_2 \in \textbf{Twk}$ $(q_t = 2)$; time complexity $t = \mathcal{O}(N^5)$

Comments:

1. The adaptive choice of messages is required.
2. We assume that the adversary can control the choice of $t \in \textbf{Twk}$. The attack does not work if the adversary does not have complete control over t. Partial truncation of the tweak can be applied to prevent the threat.

Short summary of attacks: 4

Year: 2018

Threat: Recovery of multiple messages $m_1, \dots, m_p$ **generic Feistel network**

Resources: $q_t = \mathcal{O}(N^{r-4}(n \cdot N + p))$ different tweaks, number of plaintexts per tweak: $q_e = \mathcal{O}(n \cdot N)$, time complexity $t = \mathcal{O}(n \cdot N^{r-2}(n + p))$

Comments:

1. The attack is not adaptive; only the knowledge of plaintexts is required.
2. It is assumed that the adversary knows ciphertexts for τ known plaintexts $x_1, \dots x_\tau$ and for p messages (plaintexts) under attack $m_1, \dots m_p$ for q different tweaks.
3. It is assumed that right halves of $x_1, \dots, x_\tau$ comprise all possible right halves of messages.
4. The correlation between $x_1, \dots, x_\tau$ and $m_1, \dots m_p$ is not required.

Short summary of attacks: 5

Year: 2019

Threat: Entire codebook recovery for t_1, t_2 for **FF3**.

Resources: $q_e = \mathcal{O}(N^{\frac{11}{6}})$ encryption queries on two tweaks $t_1, t_2 \in \mathsf{Twk}, q_t = 2$; time complexity $t = \mathcal{O}(N^{\frac{17}{6}})$

Comments:

1. The attack is the strengthened version of the first attack on FF3.
2. The adaptive choice of messages is required.
3. The attack does not work if the adversary cannot obtain full control over t.

Recent attack on Feistel Networks (2020)

Algorithm	Resourses	Threat
FF1, $klen = 128, r = 10$	$q = 2^{60}, t = 2^{70}$	Distinguishing attack
FF3-1, $klen = 128, r = 8$	$q = 2^{80}, t = 2^{100}$	Distinguishing attack
FEA-2, $klen = 128, r = 18$	$q = 2^{80}, t = 2^{84}$	Distinguishing attack
FEA-2, $klen = 256, r = 24$	$q = 2^{80}, t = 2^{84}$	Distinguishing attack
FEA-1, $klen = 192, r = 14$	$q = 2^{36}, t = 2^{136}$	Key recovery
FEA-1, $klen = 256, r = 16$	$q = 2^{48}, t = 2^{136}$	Key recovery
Generic Feistel network	$q = 2^{n(r-4)}, t = 2^{n(r-3)}$	Distinguishing attack

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Tiny domains: OK (provably secure shuffling methods);

Small domains: no standardized provably secure solution so far, all NIST and ISO candidates are (theoretically) broken.

Quasigroup based FPE

Quasigroup

Definition (Quasigroup)
A set Q with a binary operation on it:

$$\circ : Q \times Q \to Q,$$

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Equivalently, operations of left and right multiplication

$$L_a : Q \to Q,\ L_a(x) = a \circ x$$

$$R_a : Q \to Q,\ R_a(y) = y \circ a$$

are bijections on Q.

Pseudorandom permutations on Q

We want to measure how close the composition of quasigroup operations (for instance, left multiplications) to the random permutation on Q.

Algorithm 9 Experiment Left

function INIT(λ)
    $\pi \leftarrow^{R} Perm(Q)$
function $\mathcal{O}(m)$
    return $\pi(m)$

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$\textbf{function}\ \mathcal{O}(m)$

$\quad \textbf{return}\ \pi(m)$

Algorithm 12 Experiment Right

$\textbf{function}\ \textsc{Init}(\lambda)$

$\quad k_1, \ldots, k_\lambda \xleftarrow{R} Q$

$\textbf{function}\ \mathcal{O}(m)$

$\quad \textbf{return}\ k_1 \circ (k_2 \circ (\ldots (k_\lambda \circ m) \ldots)$

Adversary

An adversary $\mathcal{A}$ tries to distinguish between random and «structured» permutation.

$$Adv_Q^{PRP}(\mathcal{A}) = \mathbb{P}[Right(\mathcal{A}) \to 1] - \mathbb{P}[Left(\mathcal{A}) \to 1].$$

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where $A(t, q)$ is a set of adversaries, whose running time does not exceed t and who uses no more than q oracle queries.

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- We want it to be as small as possible for any given t, q.
- The inappropriate choice of quasigroup (i.e., $Q = \mathbb{Z}_N$) can make the problem trivial to solve.
- The problem might be hard for certain classes of quasigroups.
- For instance, for polynomially complete quasigroups: the problem of deciding whether or not an equation over such a quasigroup has a solution is NP-complete.

Quasigroup based FPE

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  1. given the key $k \in \mathbf{Keys}$ and the tweak $t \in \mathbf{Twk}$, use some keyed pseudorandom generator PRG to produce a sequence of «random-looking» and «independent» elements $q_i \in Q, i = 1, \dots, \lambda$, where λ is the parameter of the scheme and is chosen based on the quasigroup structure;

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  2. encrypt the message $m \in \mathbf{Dom}$ using quasigroup operation.

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$$m \to R_{q_\lambda}(\ldots R_{q_1}(m)\ldots) = (\ldots(m \circ q_1) \circ q_2 \ldots) \circ q_\lambda, \tag{2}$$

$$m \to D_{q_\lambda}(\ldots D_{q_1}(m)\ldots). \tag{3}$$

The operation D_{q_i} equals L_{q_i} if i-th bit of output of some random generator (for instance, based on values k and t) is equal to 0, and R_{q_i} otherwise.

Quasigroup based FPE: theorem

Theorem
Let q_t be the maximal number of different tweaks, q_e be the maximal number of encryption queries per tweak, t is the number of operations (running time). Then:

$$InSec^{TPRP}(t, q_t, q_e) \leq$$

$$\leq InSec^{PRG}(q_t, t + \lambda q_t q_e) + q_t InSec_Q^{PRP}(q_e, t + (1 + \lambda^2) q_e q_t).$$

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Conclusions

Further areas of research may include:

1. Consideration of specific classes of quasigroups as a basis for proposed cryptosystem (with an emphasis on the polynomially complete quasigroups);
2. Estimating the hardness of quasigroup problem based on existing results on NP-completeness;
3. Implementing the cryptosystem over specific quasigroups and estimating statistical properties of resulting algorithms;

Summary

In this report:

1. FPE: statement of the problem;
2. Proposed solutions;
3. Attacks;
4. Quasigroup based FPE;