

# Towards post-quantum cryptographic standards

## focus on code-based cryptography

Jean-Christophe Deneuville

[<jean-christophe.deneuville@enac.fr>](mailto:jean-christophe.deneuville@enac.fr)

June 2021, the 4<sup>th</sup>



The 10th Workshop on  
«Current Trends in Cryptology» CTCrypt 2021



# Outline

- 1 NIST's PQC standardization process
- 2 Recalls on coding theory
- 3 McEliece and Niederrieter: historical code-based encryption constructions
- 4 Best-known attacks
- 5 Recent code-based encryption proposals
- 6 Comparison of last CBC candidates to NIST PQC standardization
- 7 Conclusions



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- 6 Comparison of last CBC candidates to NIST PQC standardization
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# NIST PQC standardization process

**NIST** National Institute of Standards and Technology





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- Asks for post-quantum cryptographic algorithms
- 3 categories :
  - Encryption
  - Key exchange
  - Signature



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  - Error correcting codes,
  - Lattices,
  - Multivariate,
  - Hash functions,
  - Elliptic curves isogenies,
  - ...



# NIST PQC standardization process



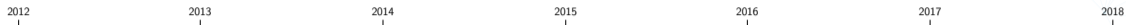
National Institute of Standards and Technology

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  - ...

security level I	At least as hard to break as AES128 (exhaustive key search)
security level II	At least as hard to break as SHA256 (collision search)
security level III	At least as hard to break as AES192 (exhaustive key search)
security level IV	At least as hard to break as SHA384 (collision search)
security level V	At least as hard to break as AES256 (exhaustive key search)



# Timeline NIST





# Timeline NIST





# Timeline NIST

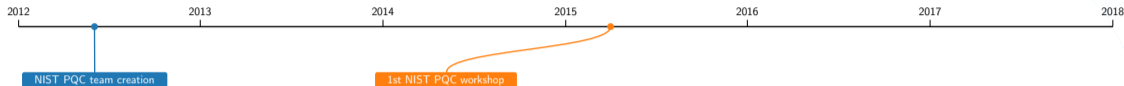
2012 2013 2014 2015 2016 2017 2018

NIST PQC team creation



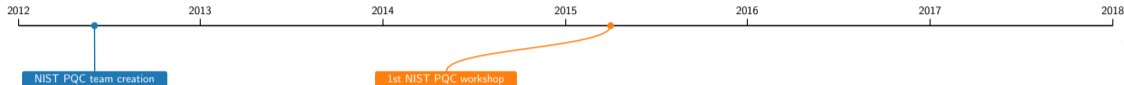


# Timeline NIST





# Timeline NIST



## Workshop on Cybersecurity in a Post-Quantum World

The advent of practical quantum computing will break all commonly used public key cryptographic algorithms. In response, NIST is researching cryptographic algorithms for public key-based key agreement and digital signatures that are not susceptible to cryptanalysis by quantum algorithms. NIST is holding this workshop to engage academic, industry, and government stakeholders. The Post Quantum Workshop will be held on April 2-3, 2015, immediately following the [2015 International Conference on Practice and Theory of Public-Key Cryptography](#). NIST seeks to discuss issues related to post-quantum cryptography and its potential future standardization.



# Timeline NIST



# Timeline NIST



## Commercial National Security Algorithm Suite

Currently, Suite B cryptographic algorithms are specified by the National Institute of Standards and Technology (NIST) and are used by NSA's Information Assurance Directorate in solutions approved for protecting classified and unclassified National Security Systems (NSS). Below, we announce preliminary plans for transitioning to quantum resistant algorithms.

### Background

IAD will initiate a transition to quantum resistant algorithms in the not too distant future. Based on experience in deploying Suite B, we have determined to start planning and communicating early about the upcoming transition to quantum resistant algorithms. Our ultimate goal is to provide cost effective security against a potential quantum computer. We are working with partners across the USG, vendors, and standards bodies to ensure there is a clear plan for getting a new suite of algorithms that are developed in an open and transparent manner that will form the foundation of our next Suite of cryptographic algorithms.



# Timeline NIST



# Timeline NIST



## NISTIR 8105

### Report on Post-Quantum Cryptography

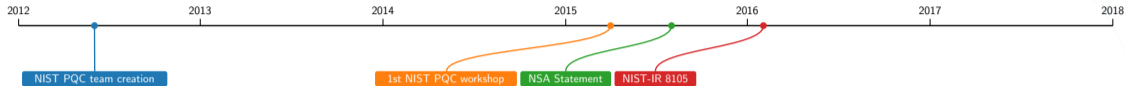
Lily Chen  
 Stephen Jordan  
 Yi-Kai Liu  
 Dustin Moody  
 Rene Peralta  
 Ray Perlner  
 Daniel Smith-Tone

## Abstract

In recent years, there has been a substantial amount of research on quantum computers – machines that exploit quantum mechanical phenomena to solve mathematical problems that are difficult or intractable for conventional computers. If large-scale quantum computers are ever built, they will be able to break many of the public-key cryptosystems currently in use. This would seriously compromise the confidentiality and integrity of digital communications on the Internet and elsewhere. The goal of *post-quantum cryptography* (also called quantum-resistant cryptography) is to develop cryptographic systems that are secure against both quantum and classical computers, and can interoperate with existing communications protocols and networks. This Internal Report shares the National Institute of Standards and Technology (NIST)'s current understanding about the status of quantum computing and post-quantum cryptography, and outlines NIST's initial plan to move forward in this space. The report also recognizes the challenge of moving to new cryptographic infrastructures and therefore emphasizes the need for agencies to focus on crypto agility.



# Timeline NIST



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### Report on Post-Quantum Cryptography

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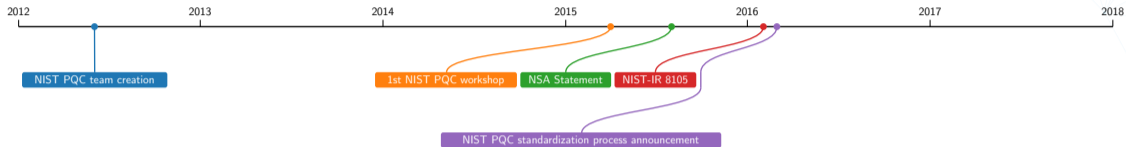
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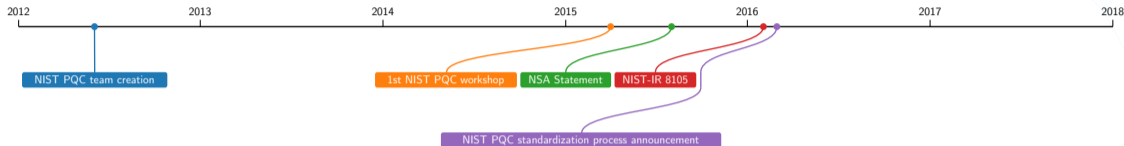
This Internal Report shares the National Institute of Standards and Technology (NIST)'s current understanding about the status of quantum computing and post-quantum cryptography, and outlines NIST's initial plan to move forward in this space. The report also recognizes the challenge of moving to new cryptographic infrastructures and therefore emphasizes the need for agencies to focus on crypto agility.



# Timeline NIST



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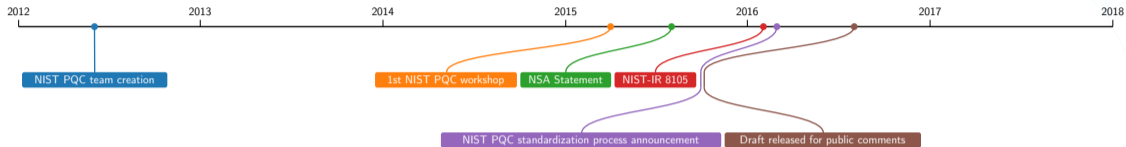
**NEWS**

## NIST Kicks Off Effort to Defend Encrypted Data from Quantum Computer Threat

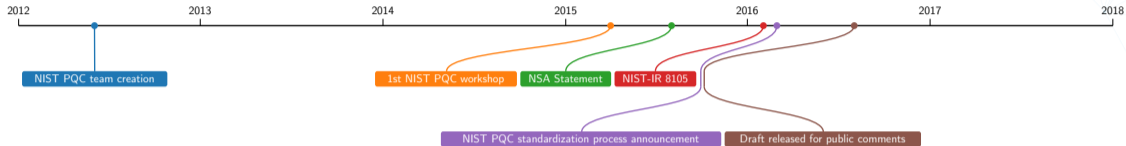
April 28, 2016

"We're looking to replace three NIST cryptographic standards and guidelines that would be the most vulnerable to quantum computers," Moody said, referring to [FIPS 186-4](#), [NIST SP 800-56A](#) and [NIST SP 800-56B](#). "They deal with encryption, key establishment and digital signatures, all of which use forms of public key cryptography."

# Timeline NIST



# Timeline NIST



Moody, Dustin (Fed)

à pqc-...@nist.nist.gov

Aug. 2019, 1st, 14:47:39 ☆ ⏪ ⋮

The National Institute of Standards and Technology (NIST) is requesting comments on a new process to solicit, evaluate, and standardize one or more quantum-resistant public-key cryptographic algorithms. Currently, public-key cryptographic algorithms are specified in FIPS 186-4, Digital Signature Standard, as well as special publications SP 800-56A Revision 2, Recommendation for Pair-Wise Key Establishment Schemes Using Discrete Logarithm Cryptography and SP 800-56B Revision 1, Recommendation for Pair-Wises Key-Establishment Schemes Using Integer Factorization Cryptography. However, these algorithms are vulnerable to attacks from large-scale quantum computers (see NISTIR 8105 Report on Post Quantum Cryptography <<http://dx.doi.org/10.6028/NIST.IR.8105>>).

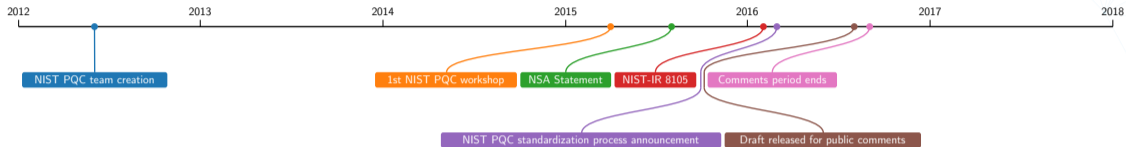
As a first step in this process, NIST is publishing draft minimum acceptability requirements, submission requirements, and evaluation criteria for candidate algorithms to solicit public comment. It is intended that the new public-key cryptography standards will specify one or more additional unclassified, publicly disclosed digital signature, public-key encryption, and key-establishment algorithms that are available worldwide, and are capable of protecting sensitive government information well into the foreseeable future, including after the advent of quantum computers.

The draft requirements and evaluation criteria are available on the NIST Computer Security Resource Center website: <http://www.nist.gov/pqcrypto>. The public comment period closes on September 16, 2016. Send comments to [pqc-comments@nist.gov](mailto:pqc-comments@nist.gov) with subject line "Comment on Post-Quantum Cryptography Requirements and Evaluation Criteria."

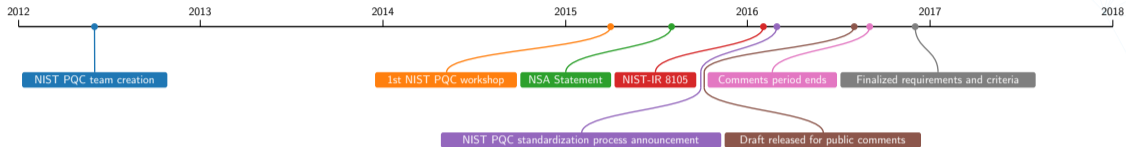
Dustin Moody  
NIST

As a reminder, you can use this forum to discuss issues related to NIST's post-quantum standardization efforts.

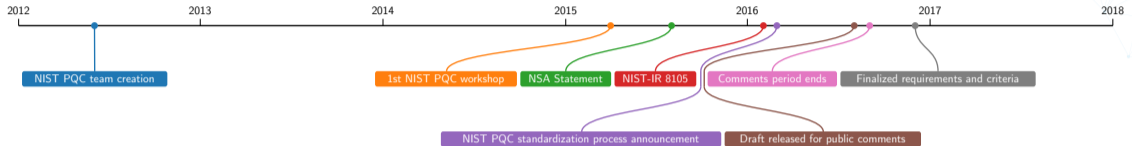
# Timeline NIST



# Timeline NIST



# Timeline NIST



Moody, Dustin (Fed)

à pqc-...@nist.nist.gov

Dec. 2016, 15th, 20:26:26 ☆ ↶ ⋮

The final submission requirements and the minimum acceptability requirements of a "complete and proper" candidate algorithm submission, as well as the evaluation criteria that will be used to appraise the candidate algorithms, can be found at <http://www.nist.gov/pqcrypto> <<http://csrc.nist.gov/groups/ST/post-quantum-crypto/index.html>>. Nominations for post-quantum candidate algorithms may now be submitted, up until the final deadline of November 30, 2017. Complete instructions on how to submit a candidate package are posted at <http://www.nist.gov/pqcrypto> <<http://csrc.nist.gov/groups/ST/post-quantum-crypto/index.html>>.

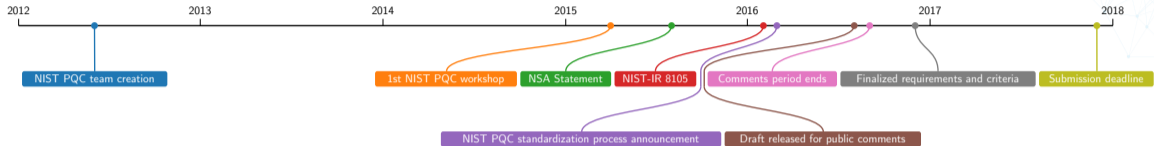
Dustin Moody

NIST

<https://csrc.nist.gov/csrc/media/projects/post-quantum-cryptography/documents/call-for-proposals-final-dec-2016.pdf>

## Submission Requirements and Evaluation Criteria for the Post-Quantum Cryptography Standardization Process

# Timeline NIST



Moody, Dustin (Fed)

à pqc-...@nist.nist.gov

Dec. 2016, 15th, 20:26:26 ☆ ↶ ⋮

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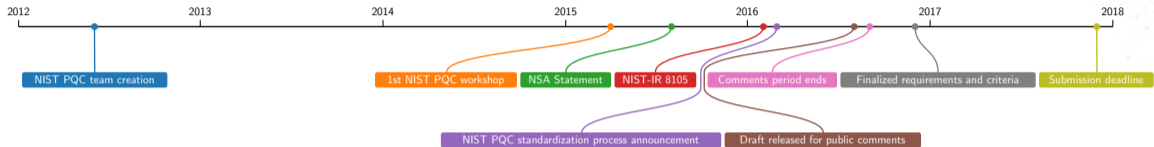
Dustin Moody

NIST

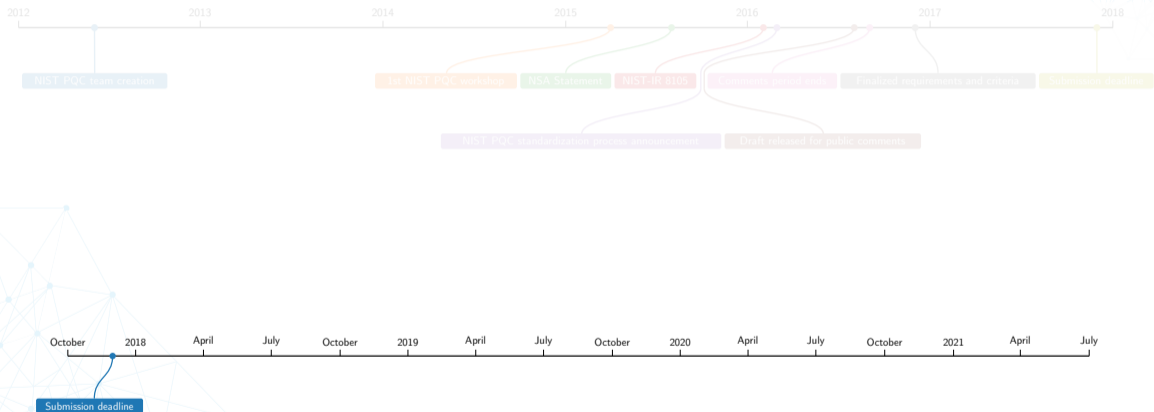
<https://csrc.nist.gov/csrc/media/projects/post-quantum-cryptography/documents/call-for-proposals-final-dec-2016.pdf>

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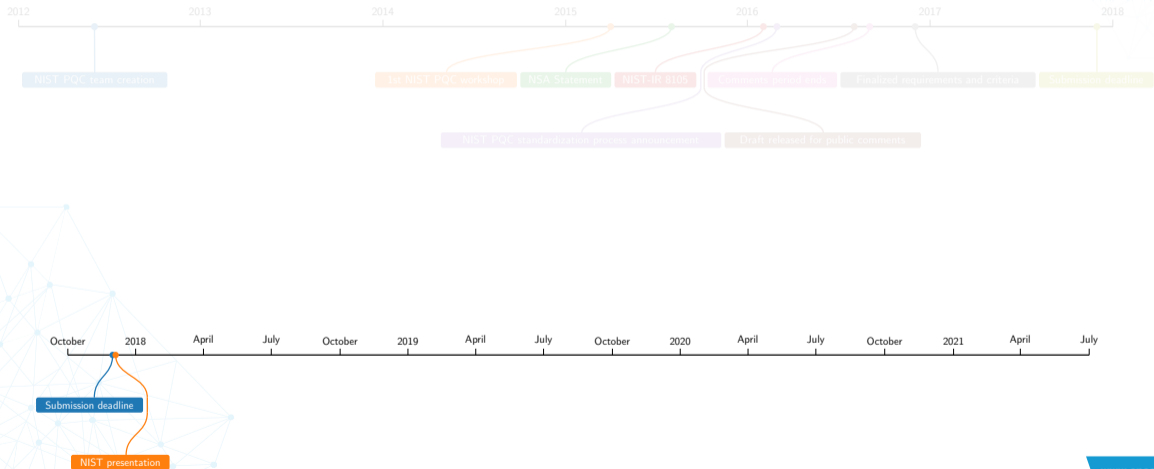
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# Timeline NIST



## AsiaCrypt'17



### THE SHIP HAS SAILED

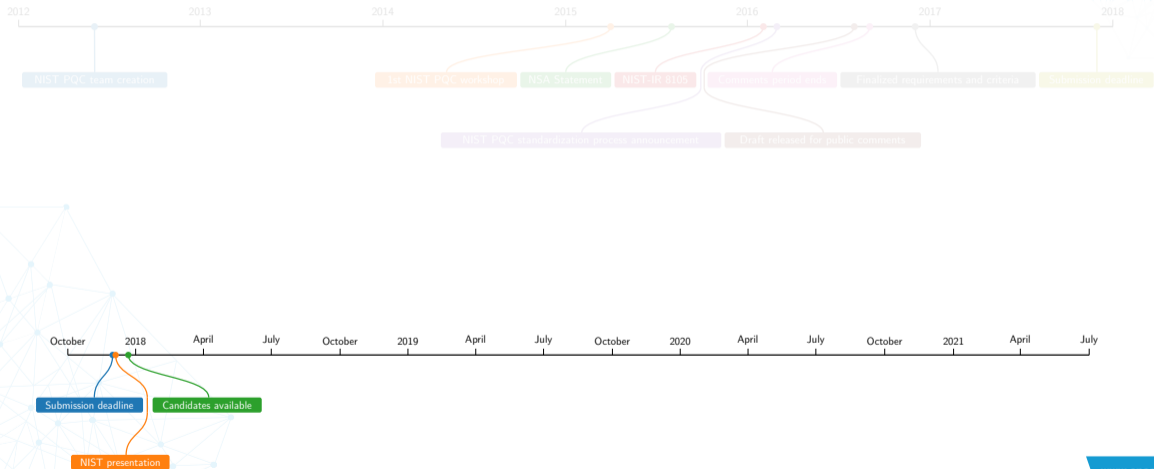
The NIST Post-Quantum Crypto "Competition"

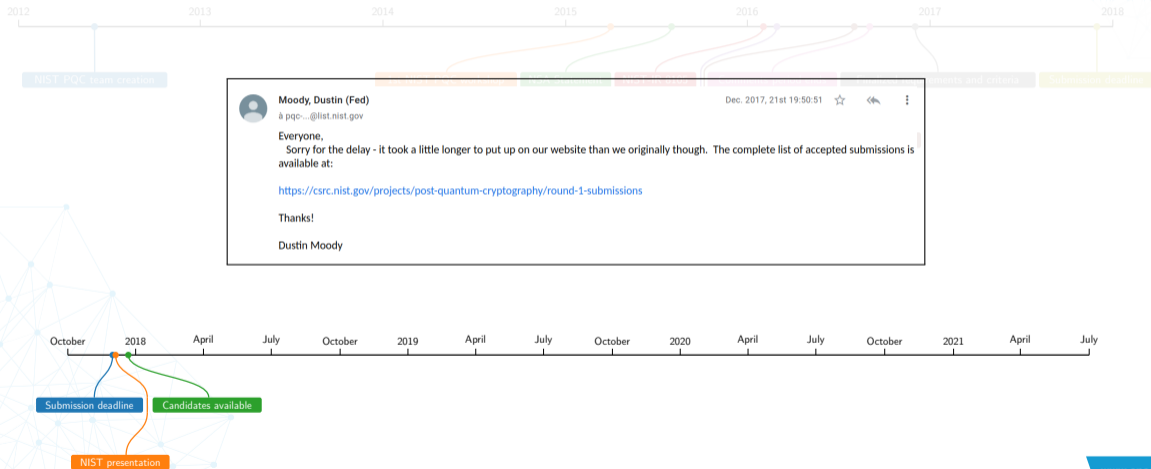
Dustin Moody, NIST

**NIST**  
National Institute of  
Standards and Technology  
U.S. Department of Commerce

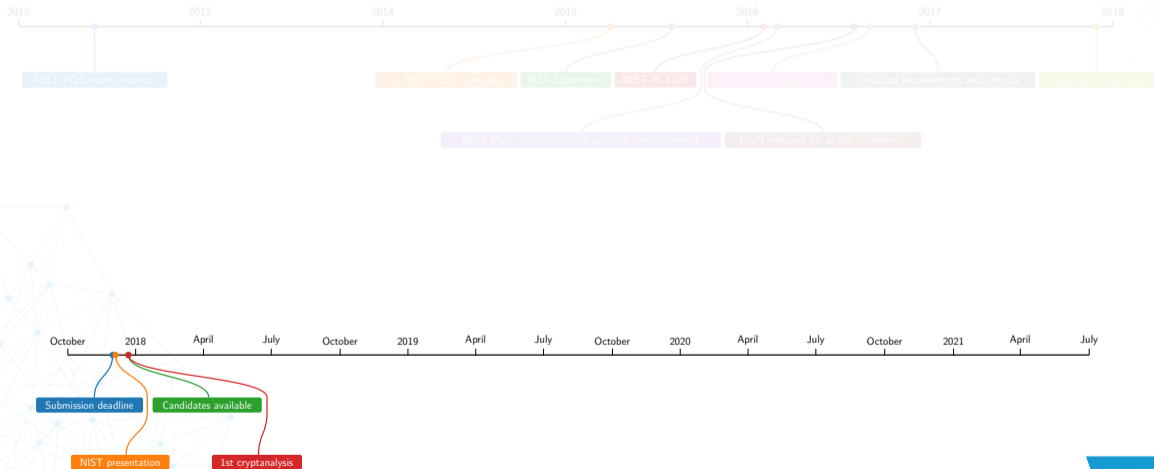


# Timeline NIST

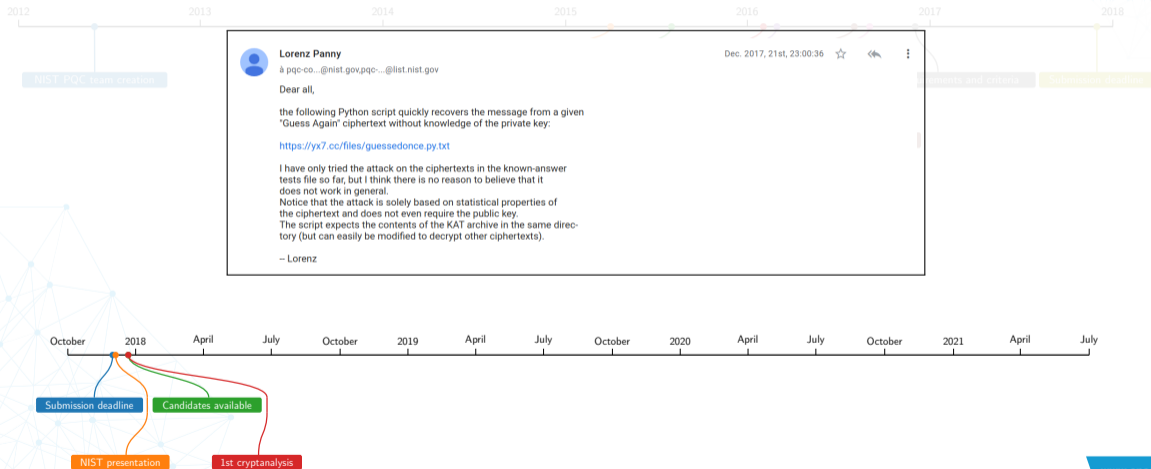




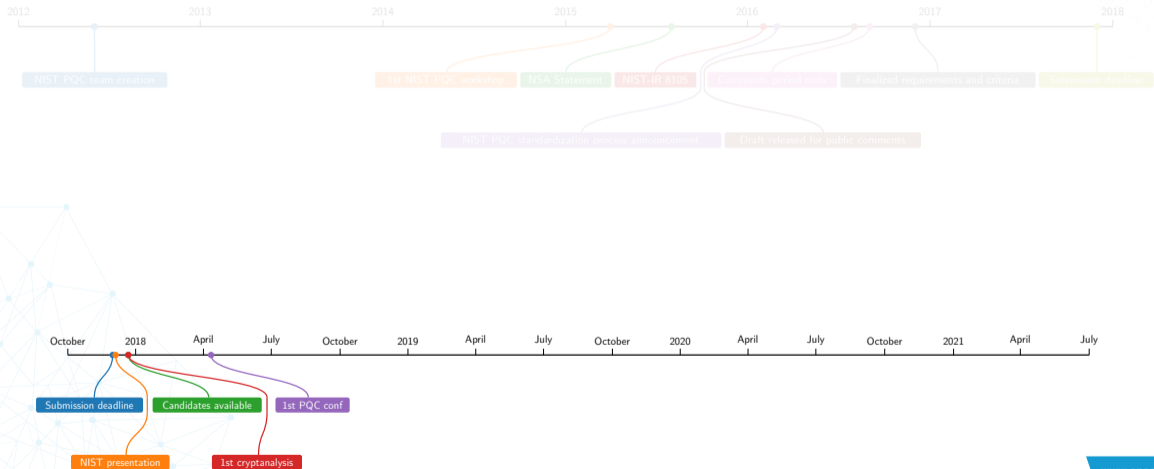
# Timeline NIST



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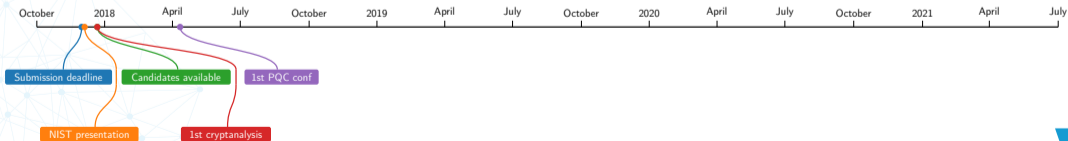


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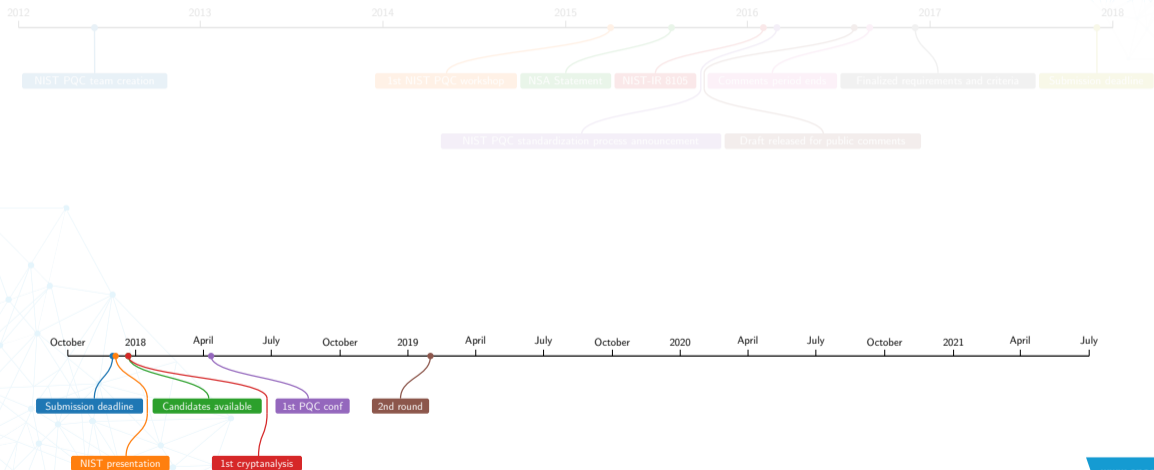




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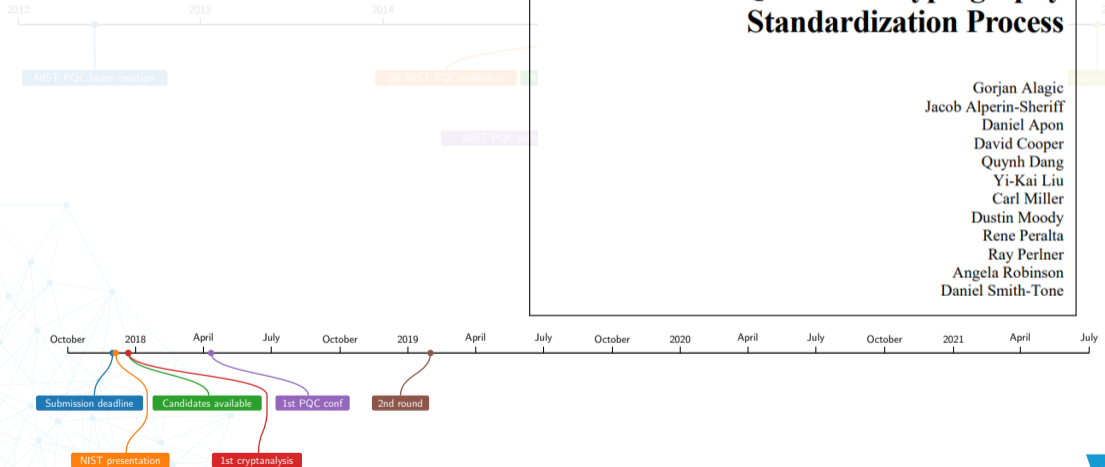


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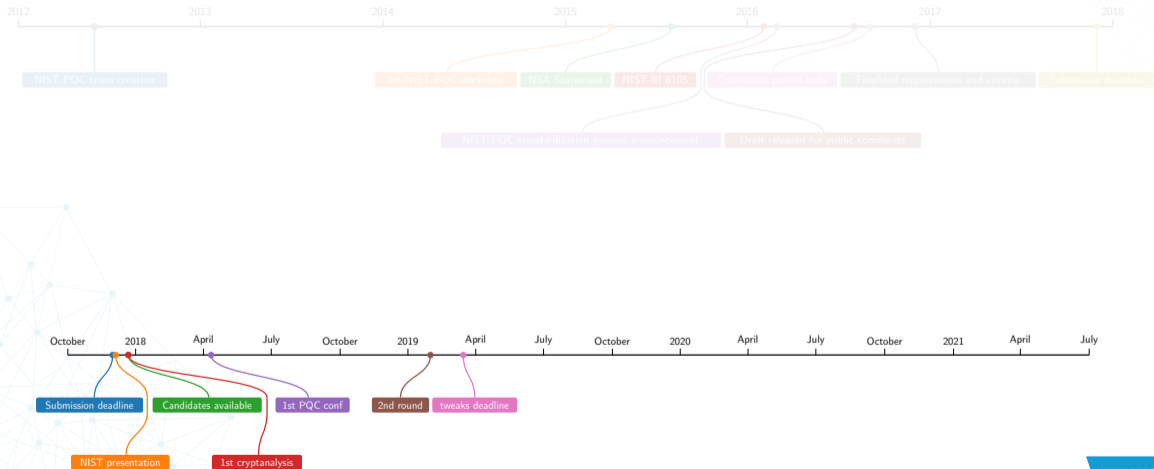


NISTIR 8240

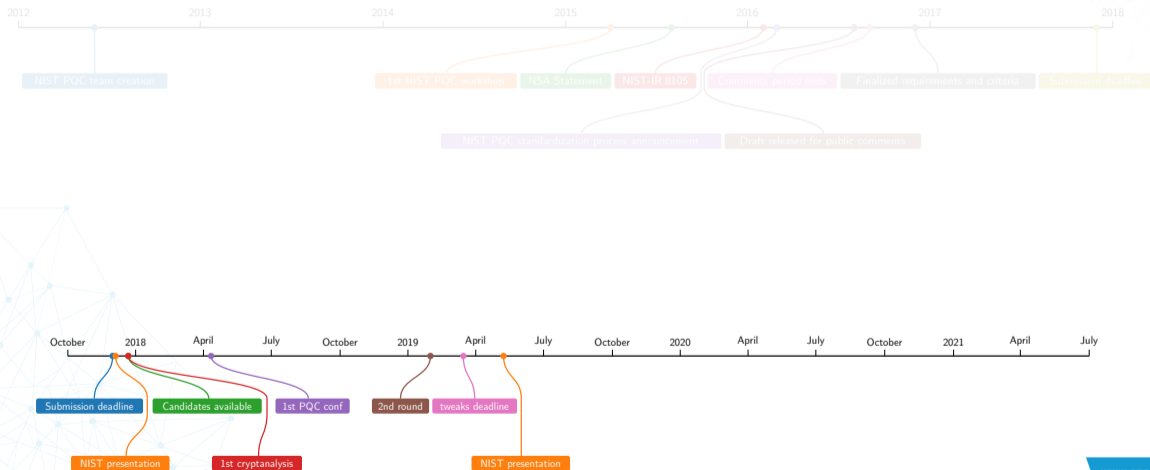
## Status Report on the First Round of the NIST Post-Quantum Cryptography Standardization Process

Gorjan Alagic  
 Jacob Alperin-Sheriff  
 Daniel Apon  
 David Cooper  
 Quynh Dang  
 Yi-Kai Liu  
 Carl Miller  
 Dustin Moody  
 Rene Peralta  
 Ray Perlner  
 Angela Robinson  
 Daniel Smith-Tone

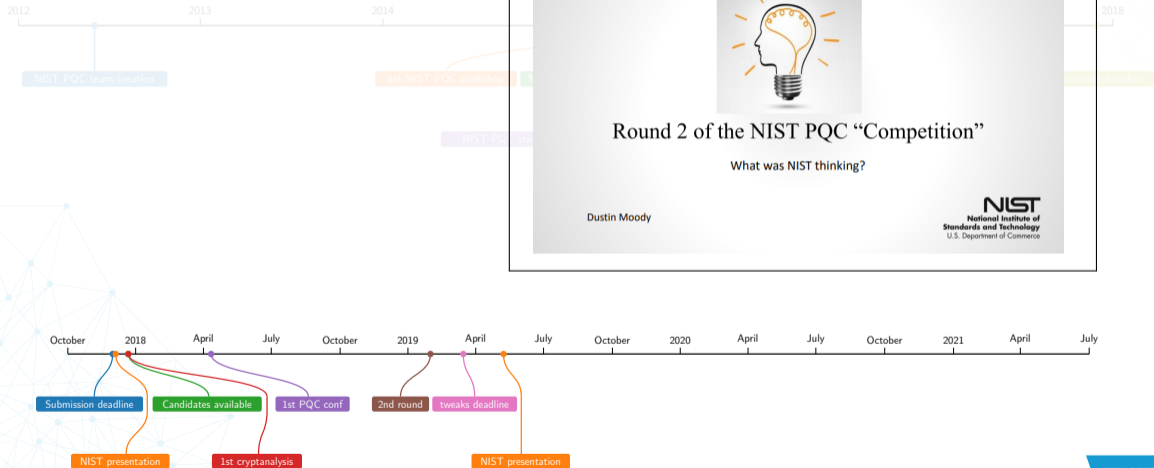
# Timeline NIST



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## PQCrypto'19



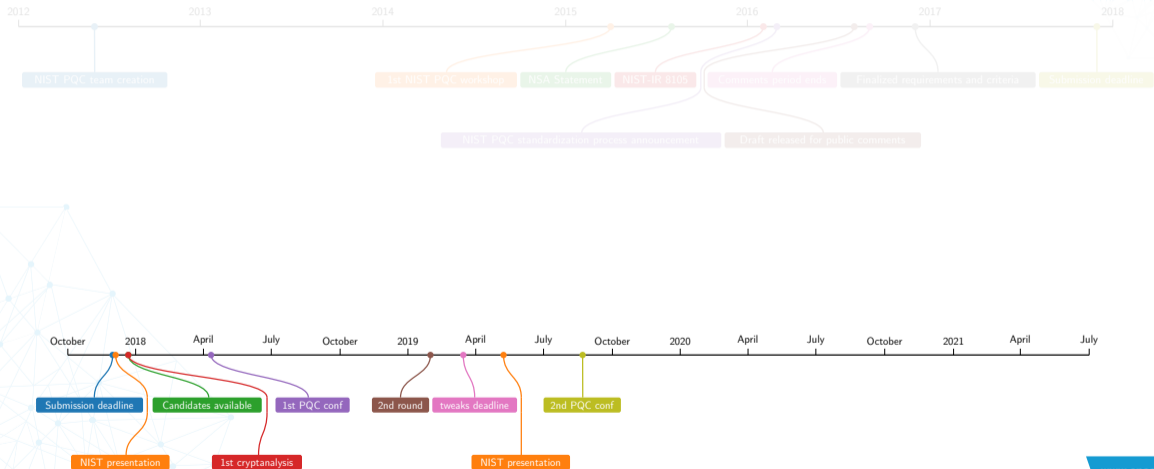
### Round 2 of the NIST PQC “Competition”

What was NIST thinking?

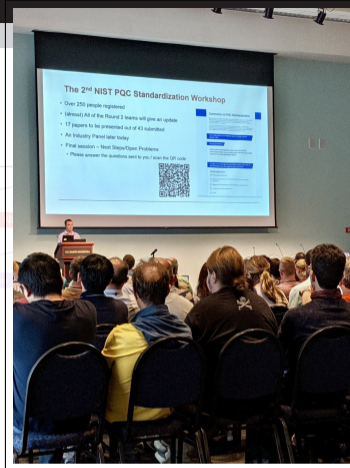
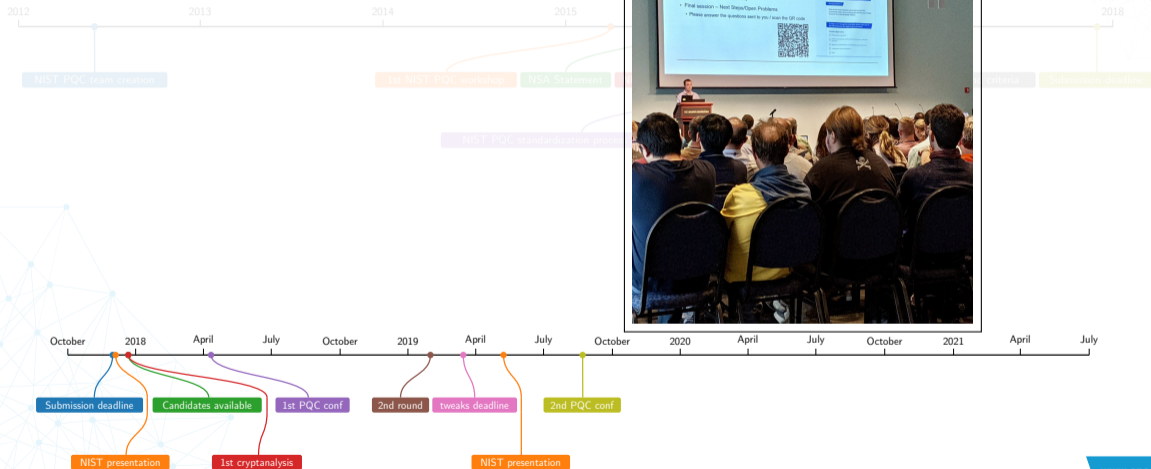
Dustin Moody

**NIST**  
National Institute of  
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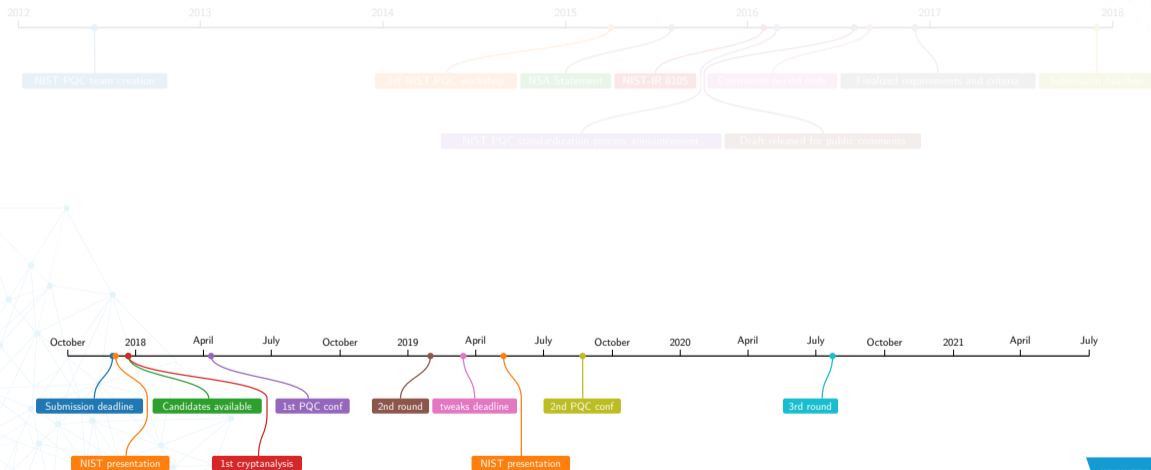
# Timeline NIST



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# Timeline NIST





# Timeline NIST

2012 2013 2014

NIST PQC team creation

1st NIST



Moody, Dustin (Fed)

à pqc-forum

Jul. 2020, 22nd, 22:51:32



## Announcement

It has been almost a year and a half since the second round of the NIST PQC Standardization Process began. After careful consideration, NIST would like to announce the candidates that will be moving on to the third round. The seven third-round Finalists are:

### Third Round Finalists

#### Public-Key Encryption/KEMs

Classic McEliece  
CRYSTALS-KYBER  
NTRU  
SABER

#### Digital Signatures

CRYSTALS-DILITHIUM  
FALCON  
Rainbow

In addition, the following eight candidate algorithms will advance to the third round:

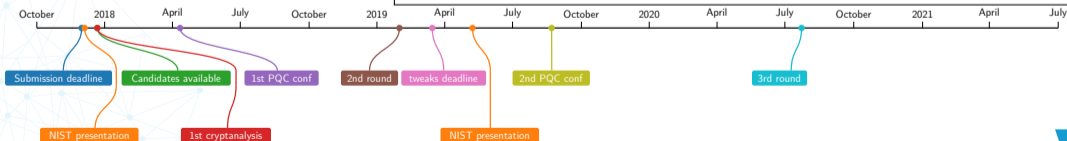
### Alternate Candidates

#### Public-Key Encryption/KEMs

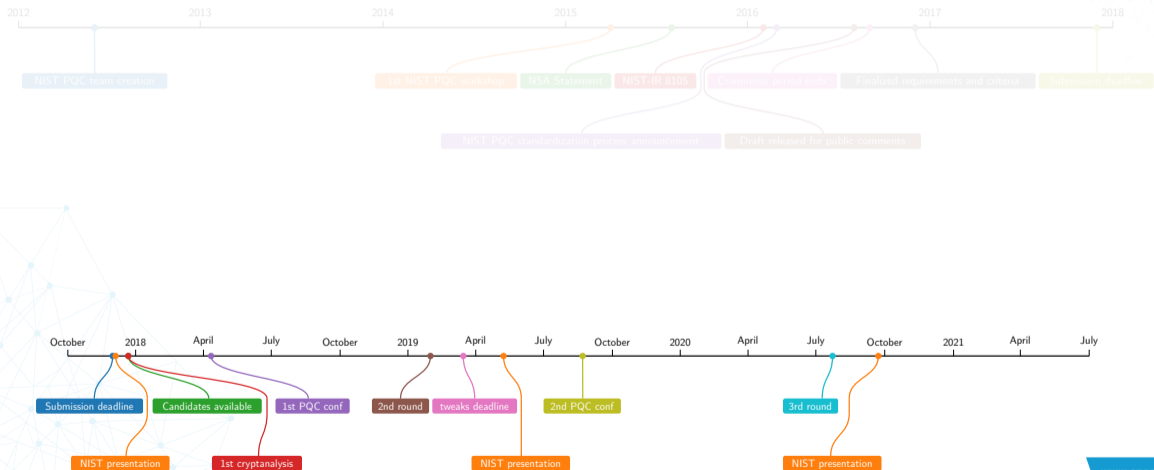
BIKE  
FrodoKEM  
HQC  
NTRU Prime  
SIKE

#### Digital Signatures

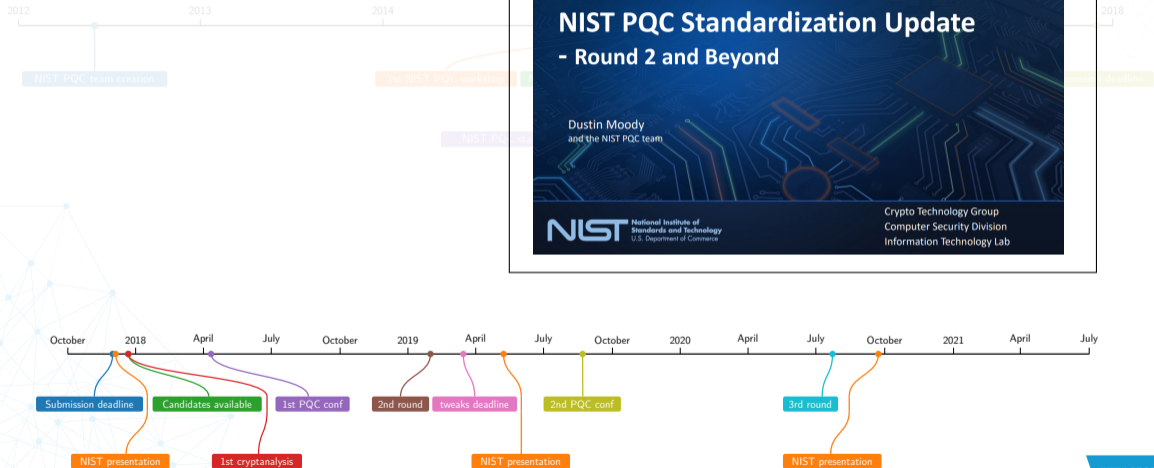
GeMSS  
Picnic  
SPHINCS+



# Timeline NIST



# Timeline NIST



PQCrypto'20

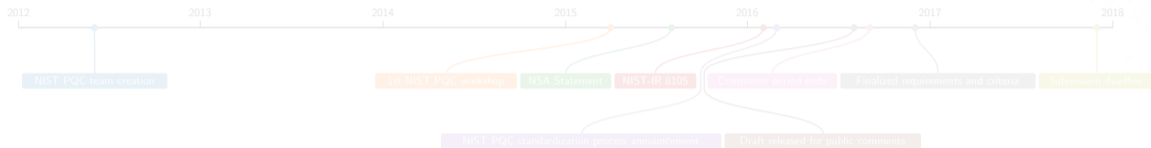
## NIST PQC Standardization Update - Round 2 and Beyond

Dustin Moody  
and the NIST PQC team

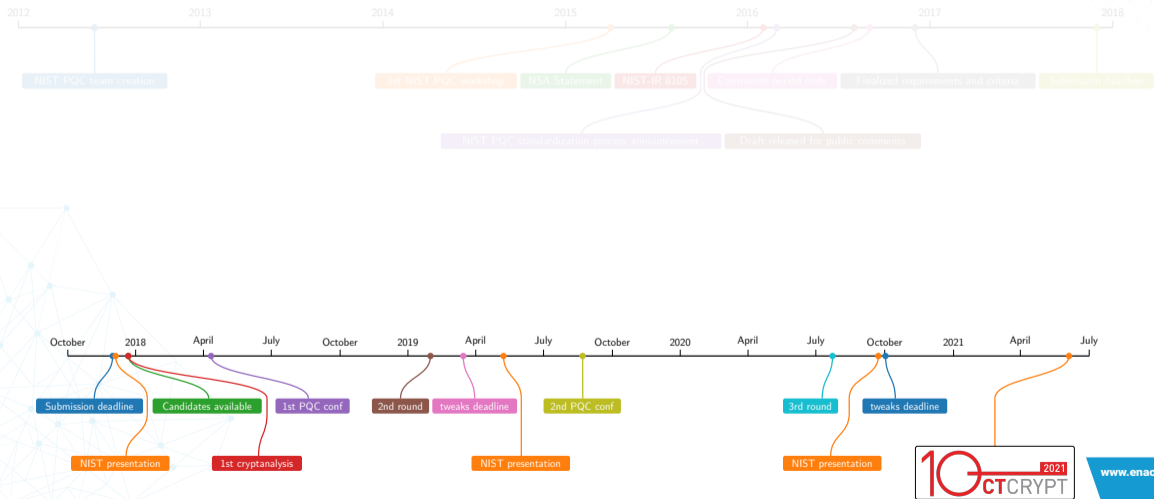
**NIST** National Institute of  
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U.S. Department of Commerce

Crypto Technology Group  
Computer Security Division  
Information Technology Lab

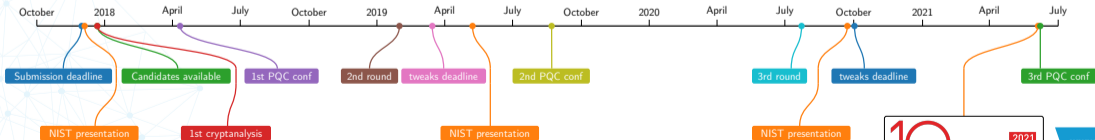
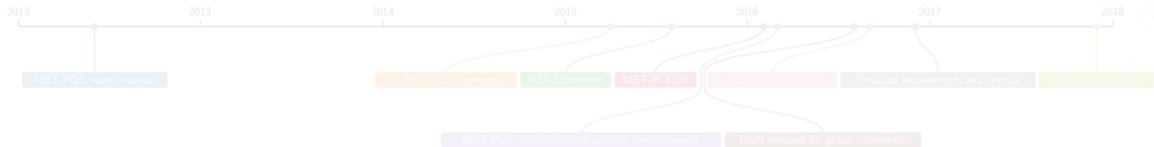
# Timeline NIST



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# Third PQC Standardization Conference

## REGISTRATION

The NIST Post-Quantum Cryptography Standardization Process has entered the third phase, in which 7 third round finalists and eight alternate candidates are being considered for standardization. NIST plans to hold a third NIST PQC Standardization Conference in June 2021 to discuss various aspects of these candidates, and to obtain valuable feedback for the final selection(s). NIST will invite each submission team of the 15 finalists and alternates to give a short update on their algorithm.

The conference will take place virtually.

### Call for Papers

- Submission deadline: **April 23, 2021**
- Notification date: **May 7, 2021**
- Conference Dates: **June 7-9, 2021**

Conference Inquiries: [pqc2021@nist.gov](mailto:pqc2021@nist.gov)

### DRAFT AGENDA

+ expand all

### Accepted Papers

## Registration Info

**Registration Fee:** \$25.00 USD

### REGISTER

The link to attend the meeting will be sent to registered attendees on **June 3, 2021**.

Registration Questions? Please contact [Crissy Robinson](#).

## EVENT DETAILS

**Starts:** June 07, 2021 - 10:00 AM EST

**Ends:** June 09, 2021 - 04:00 PM EST

**Format:** Virtual **Type:** Conference

### Agenda

**Attendance Type:** Open to public

**Audience Type:** Industry, Government, Academia, Other

## PARENT PROJECT

See: [Post-Quantum Cryptography](#)

## RELATED EVENTS

**Previous:**

<< [Second PQC Standardization Conference](#)

## RELATED TOPICS

**Security and Privacy:** [post-quantum cryptography](#)

## RELATED PAGES

**Event:** [PQC Conference 2018](#)

**News Item:** [PQC Third Round Candidate Announcement](#)

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<https://csrc.nist.gov/Events/2021/third-pqc-standardization-conference>

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See: [Post-Quantum Cryptography](#)

## RELATED EVENTS

**Previous:**

[<< Second PQC Standardization Conference](#)

## RELATED TOPICS

**Security and Privacy:** [post-quantum cryptography](#)

## RELATED PAGES

**Event:** [PQC Conference 2018](#)

**News Item:** [PQC Third Round Candidate Announcement](#)

# Overview of the candidates

primitive category	PKE / KEM	Signature	Total
Lattice-based			
Code-based			
Hash-based			
Multivariate-based			
Isogeny-based			
Other			
Total			

# Overview of the candidates

primitive category	PKE / KEM	Signature	Total
Lattice-based	22	5	27
Code-based	19	3	22
Hash-based	0	3	3
Multivariate-based	2	7	9
Isogeny-based	1	0	1
Other	5	2	7
Total	49	20	69

# Overview of the candidates

primitive category	PKE / KEM	Signature	Total
Lattice-based	22 → 9	5 → 3	27 → 12
Code-based	19 → 7	3 → 0	22 → 7
Hash-based	0 → 0	3 → 2	3 → 2
Multivariate-based	2 → 0	7 → 4	9 → 4
Isogeny-based	1 → 1	0 → 0	1 → 1
Other	5 → 0	2 → 0	7 → 0
Total	49 → 17	20 → 9	69 → 26

# Overview of the candidates

primitive category	PKE / KEM	Signature	Total
Lattice-based	$22 \rightarrow 9 \rightarrow 3 + 2$	$5 \rightarrow 3 \rightarrow 2 + 0$	$27 \rightarrow 12 \rightarrow 5 + 2$
Code-based	$19 \rightarrow 7 \rightarrow 1 + 2$	$3 \rightarrow 0 \rightarrow 0 + 0$	$22 \rightarrow 7 \rightarrow 1 + 2$
Hash-based	$0 \rightarrow 0 \rightarrow 0 + 0$	$3 \rightarrow 2 \rightarrow 0 + 2$	$3 \rightarrow 2 \rightarrow 0 + 2$
Multivariate-based	$2 \rightarrow 0 \rightarrow 0 + 0$	$7 \rightarrow 4 \rightarrow 1 + 1$	$9 \rightarrow 4 \rightarrow 1 + 1$
Isogeny-based	$1 \rightarrow 1 \rightarrow 0 + 1$	$0 \rightarrow 0 \rightarrow 0 + 0$	$1 \rightarrow 1 \rightarrow 0 + 1$
Other	$5 \rightarrow 0 \rightarrow 0 + 0$	$2 \rightarrow 0 \rightarrow 0 + 0$	$7 \rightarrow 0 \rightarrow 0 + 0$
Total	$49 \rightarrow 17 \rightarrow 4 + 5$	$20 \rightarrow 9 \rightarrow 3 + 3$	$69 \rightarrow 26 \rightarrow 7 + 8$

## 3<sup>rd</sup> round candidates

	Finalists	Alternates
PKE/KEM	Classic McEliece	BIKE
	CRYSTALS-KYBER	FrodoKEM
	NTRU	HQC
	SABER	NTRU Prime
		SIKE
Signature	CRYSTALS-DILITHIUM	GeMSS
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this talk

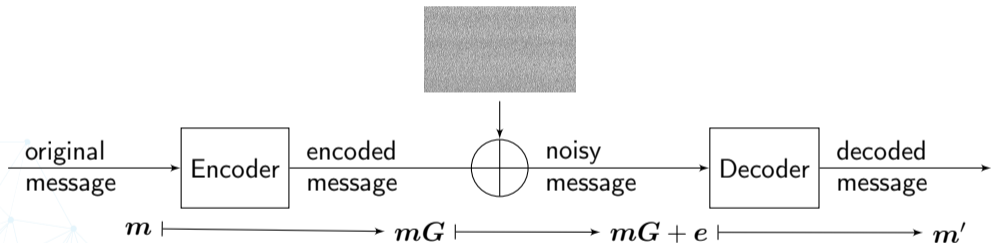


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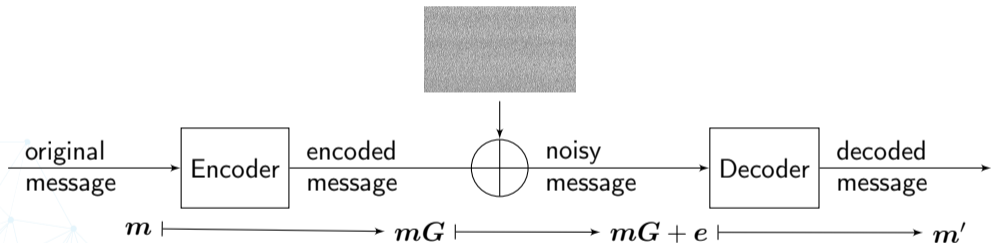
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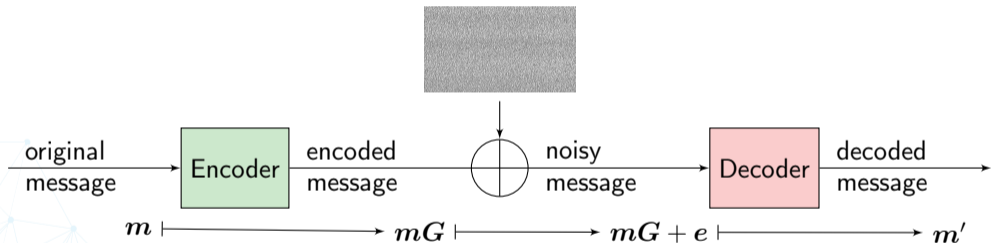


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- For code-based PKC, most of the time, **public encoder** / **private decoder**.



# Definitions

## Linear code

A *linear code* of dimension  $k$  and length  $n$  over  $\mathbb{F}_q$  is a  $k$ -dimensional subspace of  $\mathbb{F}_q^n$ .

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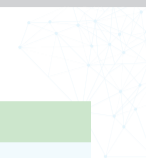
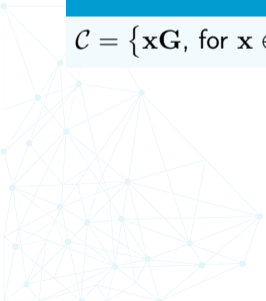
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The Hamming weight of a word  $\mathbf{u}$  is the number of its non-zero coordinates:

$$|\mathbf{u}| = \#\{i \in \{0, \dots, n-1\} \text{ such that } u_i \neq 0\}$$

$$\text{example : } |(0, 1, 0, 0, 1, 0, 1, 0)| = 3$$



# Hard problems for cryptography

## Syndrome Decoding (SD) problem

Given  $\mathbf{H} \in \mathbb{F}_q^{(n-k) \times n}$  and  $\mathbf{s} \in \mathbb{F}_q^{n-k}$ , find  $\mathbf{x} \in \mathbb{F}_q^n$  of Hamming weight  $|\mathbf{x}| \leq w$  such that:

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- The SD problem has been proved NP-complete [BMvT78]
- Hardest instances are obtained with  $w$  close to the Gilbert-Varshamov bound (essentially  $w \approx n/9$  for  $k = n/2$ )
- Best-known algorithms: Information Set Decoding (ISD), see later



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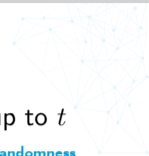




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invertible matrix  $\mathbf{S} \xleftarrow{\$} \mathbb{F}_2^{k \times k}$

permutation matrix  $\mathbf{P} \xleftarrow{\$} \mathbb{F}_2^{n \times n}$

$$\tilde{c} = \mathcal{D}_{\mathcal{C}}(c\mathbf{P}^{-1}) = \mathcal{D}_{\mathcal{C}}(m\mathbf{S}\mathbf{G} + e\mathbf{P}^{-1})$$

$$m = \tilde{c}\mathbf{S}^{-1}$$



message  $\mathbf{m} \in \mathbb{F}_2^k$

$$\text{pk} = (\tilde{\mathbf{G}} = \mathbf{S}\mathbf{G}\mathbf{P}, t)$$

$$\xrightarrow{\quad c \quad}$$

$$\xleftarrow{\quad}$$

$e \in \mathbb{F}_2^n$  such that  $|e| \leq t$

$$c = m\tilde{\mathbf{G}} + e \in \mathbb{F}_2^n$$



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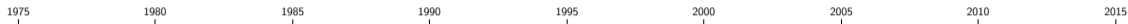
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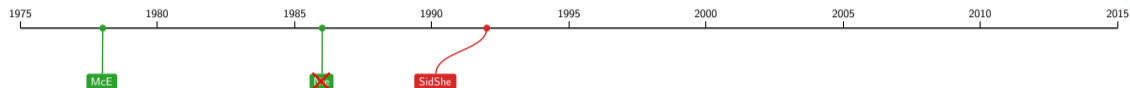
McEliece original proposal with binary Goppa codes [McE78]

# Instantiations and cryptanalyses



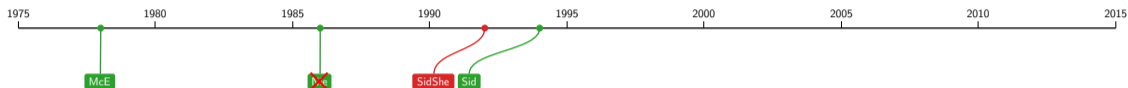
Niederreiter's (dual) approach, with GRS codes [Nie86]

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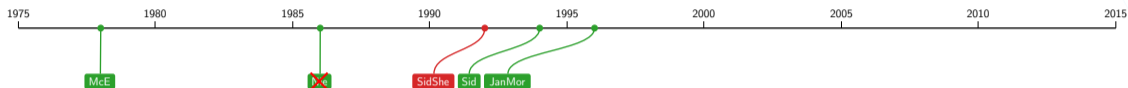
Sidelnikov Shestakov, cryptanalysis of Niederreiter's proposal [SS92]

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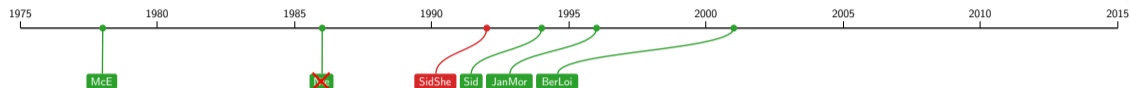
Sidelnikov proposes Reed-Muller codes [Sid94]

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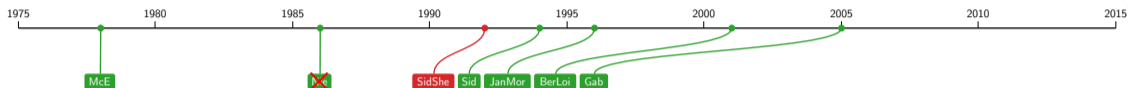
Janwa Moreno propose Alg. Geo. codes and their subfield subcodes [JM96]

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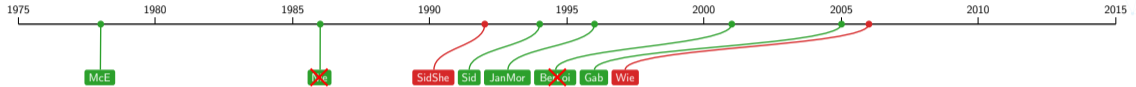
Berger Loidreau, propose subcodes of GRS codes [BL04]

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Gaborit proposes QC-BCH codes [Gab05]

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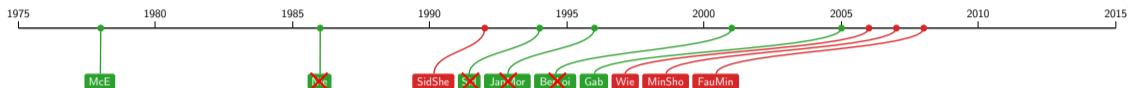
Wieschebrink's square attack:  $\mathcal{C} \star \mathcal{C}$  [Wie06]

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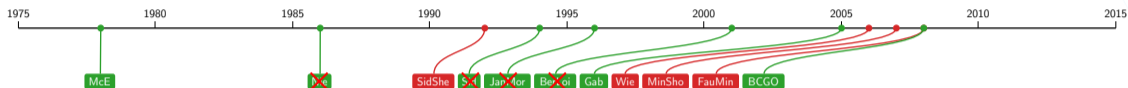
Minder Shokrollahi, subexponential time attack on RM codes [MS07]

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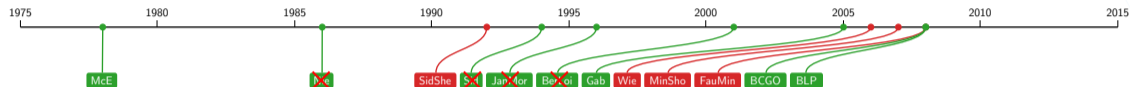
Faure Minder, attack on AG codes for genus  $\leq 2$  [FM08]

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Berger Cayrel Gaborit Otmani, propose QC alternant codes [BCGO09]

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Bernstein Lange Peters, propose  $q$ -ary "wild" Goppa codes [BLP10]

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Otmani Tillich Dallot, Attacks on QC codes [OTD10]

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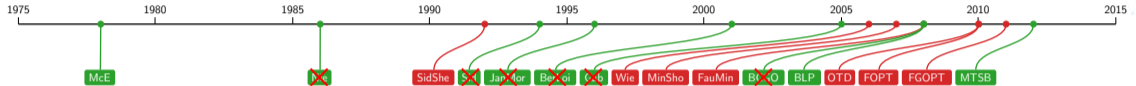
Faugère Otmani Perret Tillich, more attacks on QC codes [FOPT10]

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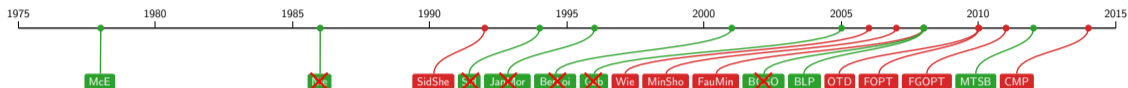
Faugère Otmani Gautier Perret Tillich, distinguisher high rate goppa codes [FGUO<sup>+</sup>13]

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Misoczki Tillich Sendrier Barreto, propose (QC-)MDPC codes [MTSB13]

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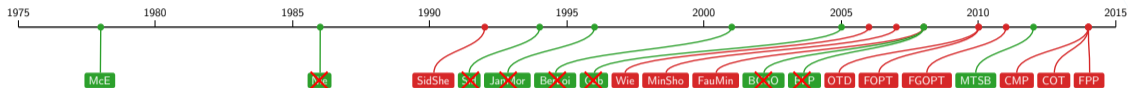
Couvreur Márquez Pellikaan, attack on AG codes [CMCP14]

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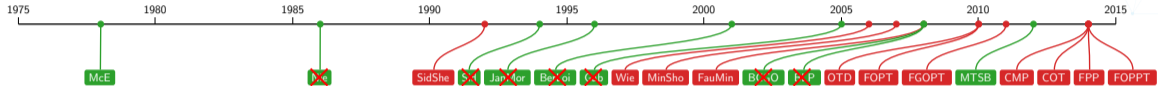
Couvreur Otmani Tillich, Goppa codes with  $m = 2$  [COT14]

# Instantiations and cryptanalyses



Faugère Perret Portzamparc, some Goppa codes with  $m = 2, 3$  [FPdP14]

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Faugère Otmani Perret Portzamparc Tillich, Further attack on QC and QD codes [FOP<sup>+</sup>16]



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Best approach to solve the SD problem: Information Set Decoding (ISD).

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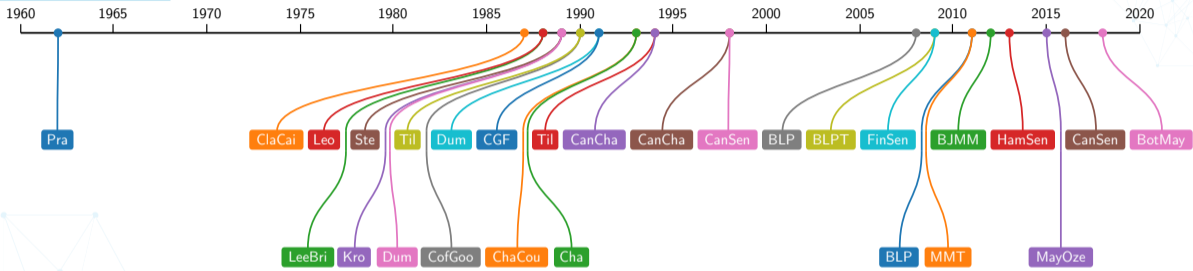
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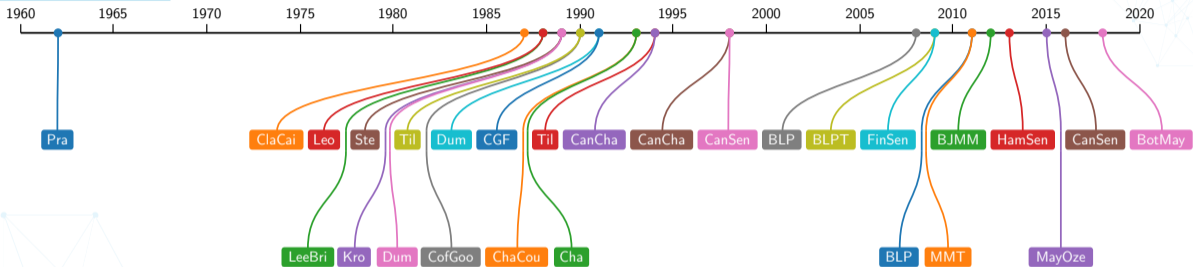
Complexity:  $\left(\frac{1}{1-\frac{k}{n}} + o(1)\right)^t$  with  $t = \Theta\left(\frac{n}{\log n}\right) \rightarrow$  pk size:  $(c + o(1)) \lambda^2 \log_2(\lambda)^2$  bits

# Information Set Decoding improvements



- References:
- [Pra62]
  - [Leo88]
  - [CG90]
  - [CGF91]
  - [vT94]
  - [CS98]
  - [FS09]
  - [BJMM12]
  - [CS16]
  - [CC81]
  - [Kro89]
  - [vT90]
  - [Cha92]
  - [CC94]
  - [BLP08]
  - [BLP11]
  - [HS13]
  - [BM18]
  - [LB88]
  - [Ste88]
  - [Dum91]
  - [CC93]
  - [CC98]
  - [BLPvT09]
  - [MMT11]
  - [MO15]

# Information Set Decoding improvements



≈ 60 years of research: same complexity, same constant in exponent, slightly improved  $o(1)$

- References:
- [Pra62]
  - [Leo88]
  - [CG90]
  - [CGF91]
  - [vT94]
  - [CS98]
  - [FS09]
  - [BJMM12]
  - [CS16]
  - [CC81]
  - [Kro89]
  - [vT90]
  - [Cha92]
  - [CC94]
  - [BLP08]
  - [BLP11]
  - [HS13]
  - [BM18]
  - [LB88]
  - [Ste88]
  - [Dum91]
  - [CC93]
  - [CC98]
  - [BLPvT09]
  - [MMT11]
  - [MO15]



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# BIKE – bit flipping key encapsulation [AAB<sup>+</sup>19]



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$h_0, h_1 \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$  with  $h_0$  invertible

$$h \leftarrow h_1 h_0^{-1}$$

$$e_0, e_1 \leftarrow \text{Bit-Flipping}(c, h_0, h_1)$$

$$\begin{array}{c} \xrightarrow{\text{pk}=(h,t)} \\ \xleftarrow{c} \end{array}$$

message  $m \in \mathbb{F}_2^k$

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Shared key derived from  $e_0, e_1$



# HQC [ABD<sup>+</sup>18]

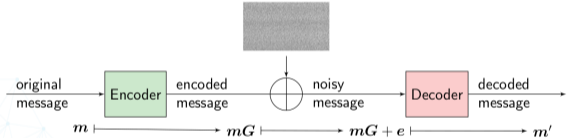


# HQC [ABD<sup>+</sup>18]



## Coding theory

Coding theory is the science of (efficiently) adding redundancy to information in order to detect/correct errors that could occur during transmission.



Preliminary remarks:


- Hopefully, we have  $m' = m$
- For code-based PKC, most of the time, **public encoder** / **private decoder**.

Code-based Cryptography / Recalls on coding theory
The French civil Aviation University
10/33

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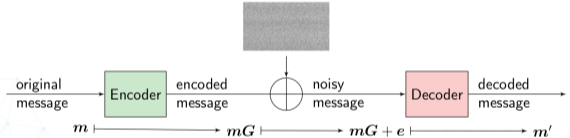
HQC uses a **public** decoder!

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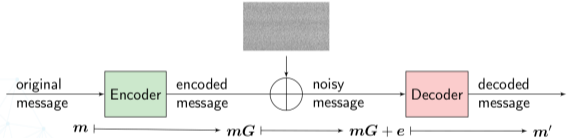
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Ecole Nationale de l'Aviation Civile



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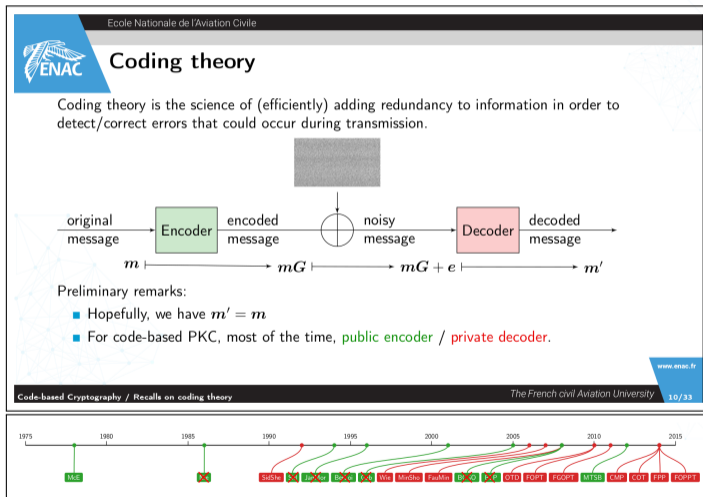
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$$\mathbf{x}, \mathbf{y} \xleftarrow{\$} \mathcal{S}_w^n(\mathbb{F}_2)$$

$$\mathbf{h} \xleftarrow{\$} \mathbb{F}_2^n, \mathbf{s} \leftarrow \mathbf{x} + \mathbf{y}\mathbf{h}$$

$$\mathbf{m} \leftarrow \mathcal{D}_{\mathcal{C}}(\mathbf{c}_0 - \mathbf{c}_1\mathbf{y})$$



$$\text{message } \mathbf{m} \in \mathbb{F}_2^k$$

$$\mathbf{e}_0, \mathbf{e}_1, \mathbf{e} \xleftarrow{\$} \mathcal{S}_t^n(\mathbb{F}_2)$$

$$\mathbf{c}_0 = \mathbf{e}_0 + \mathbf{e}_1\mathbf{h} \in \mathbb{F}_2^n$$

$$\mathbf{c}_1 = \mathbf{m}\mathbf{G} + \mathbf{s}\mathbf{e}_1 + \mathbf{e} \in \mathbb{F}_2^n$$

$$\xrightarrow{\text{pk}=(\mathbf{h}, \mathbf{s}, t)}$$

$$\xleftarrow{\mathbf{c}_0, \mathbf{c}_1}$$



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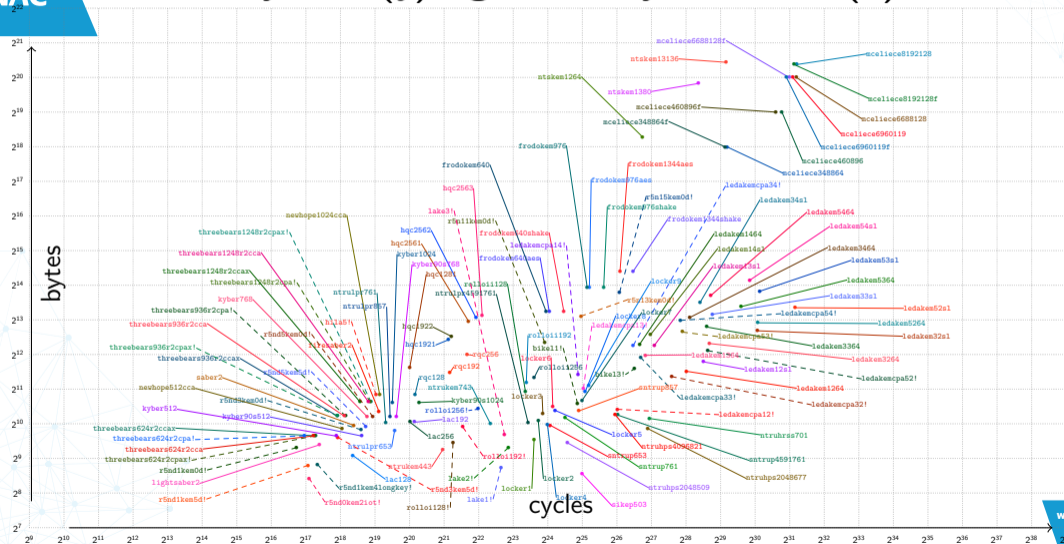
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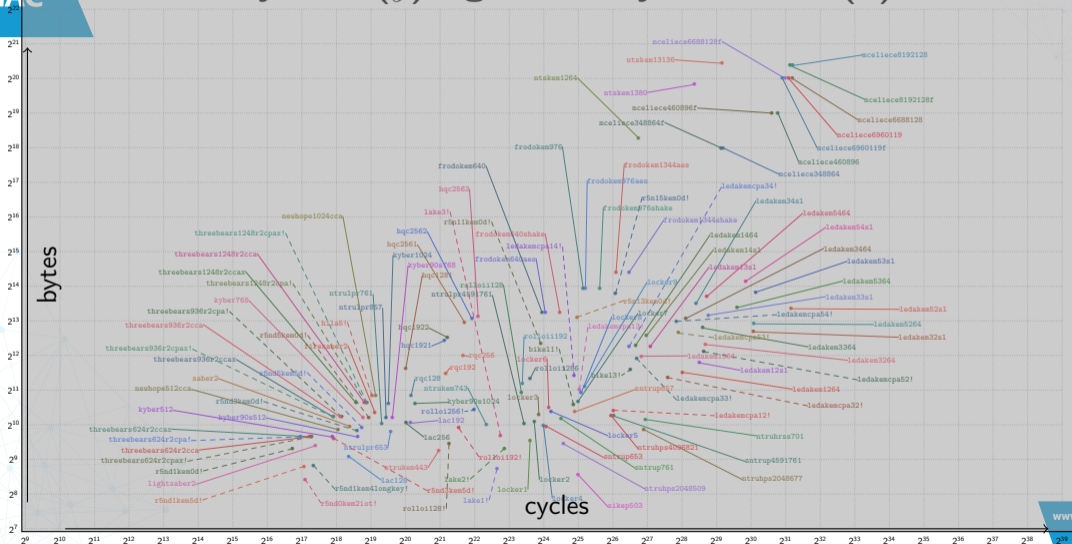
Source: supercop  
amd64; Sandy Bridge (206a7);  
2011 Intel Core i3-2310M; 2 x 2100MHz;  
date: 2020 - 06 - 18  
<https://bench.cr.yp.to/results-kem.html>

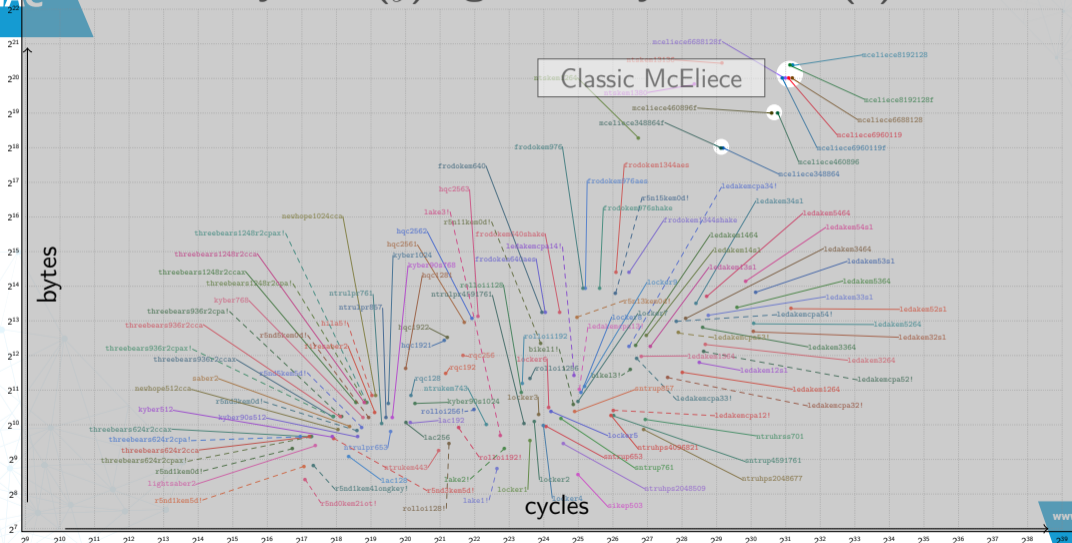


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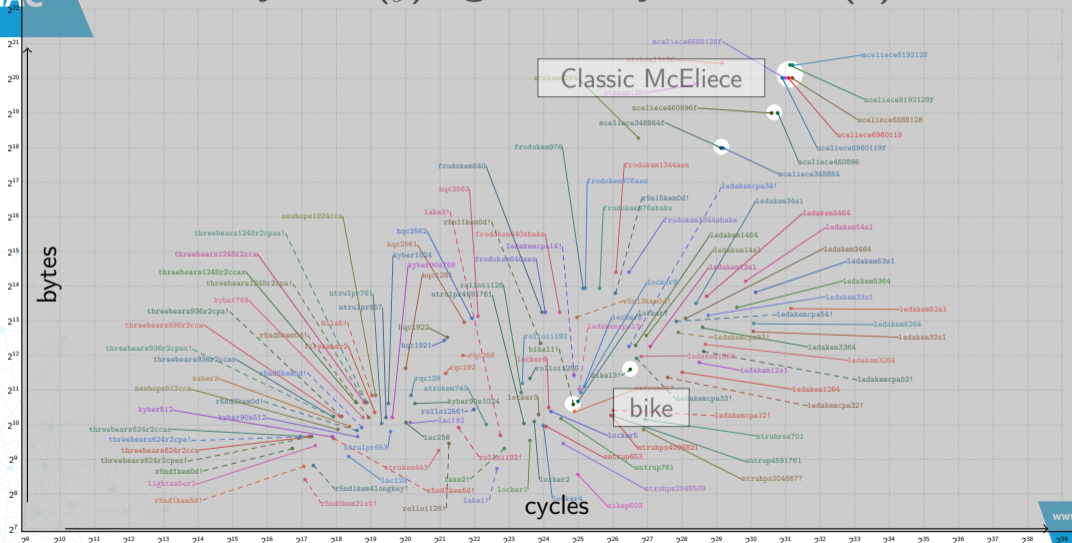
cycles

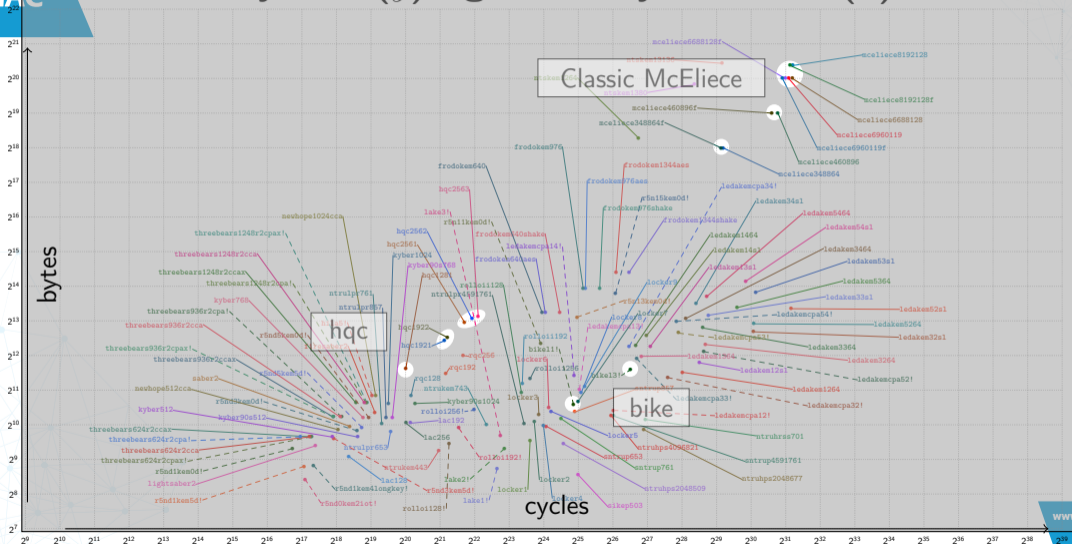






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# Ciphertext size ( $y$ ) against Encaps time ( $x$ )

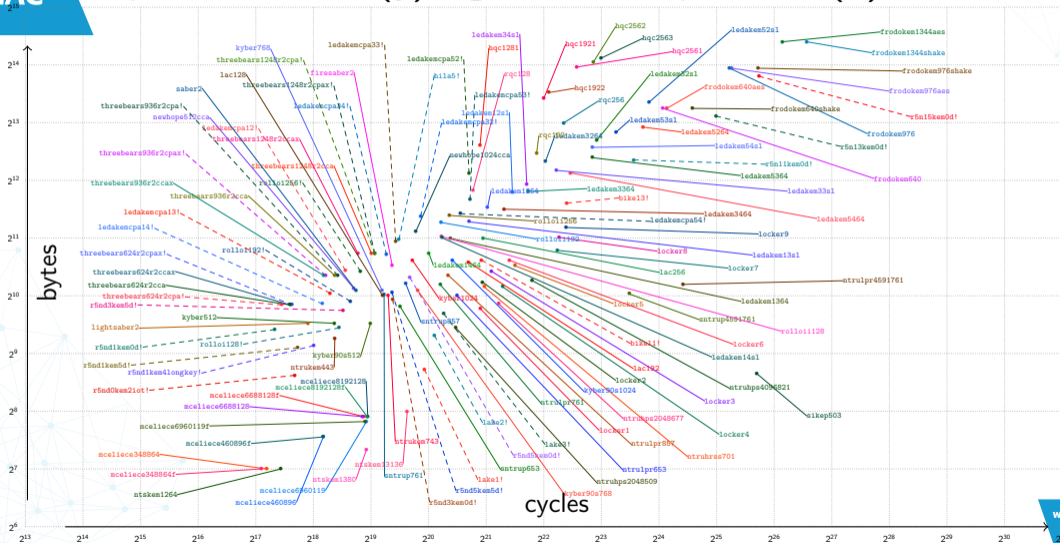
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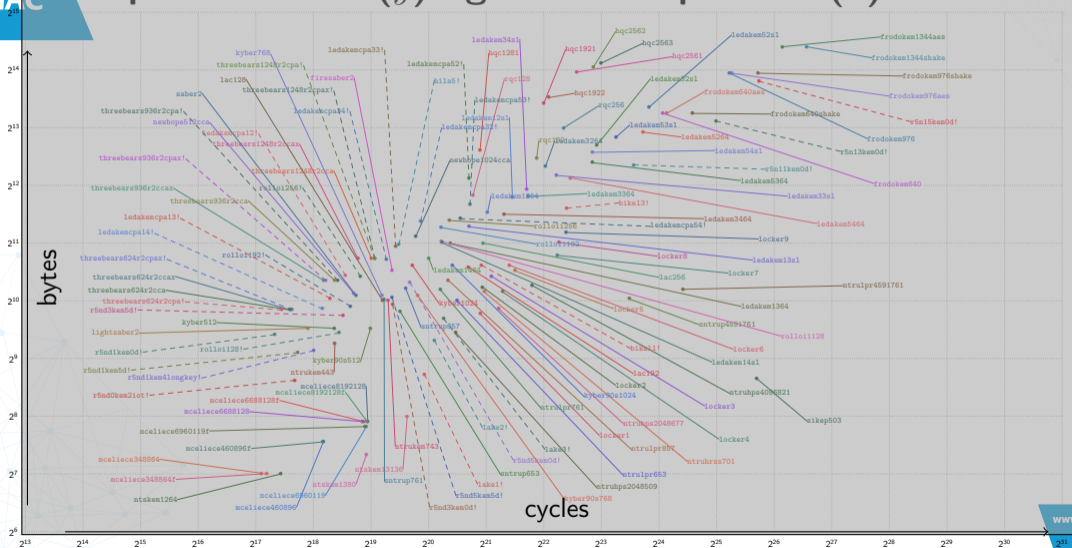


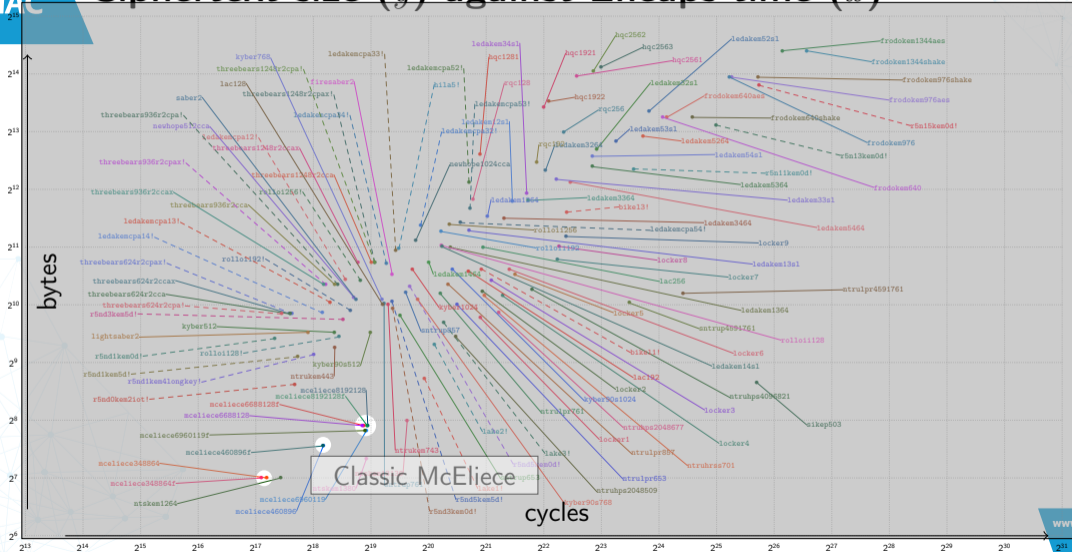
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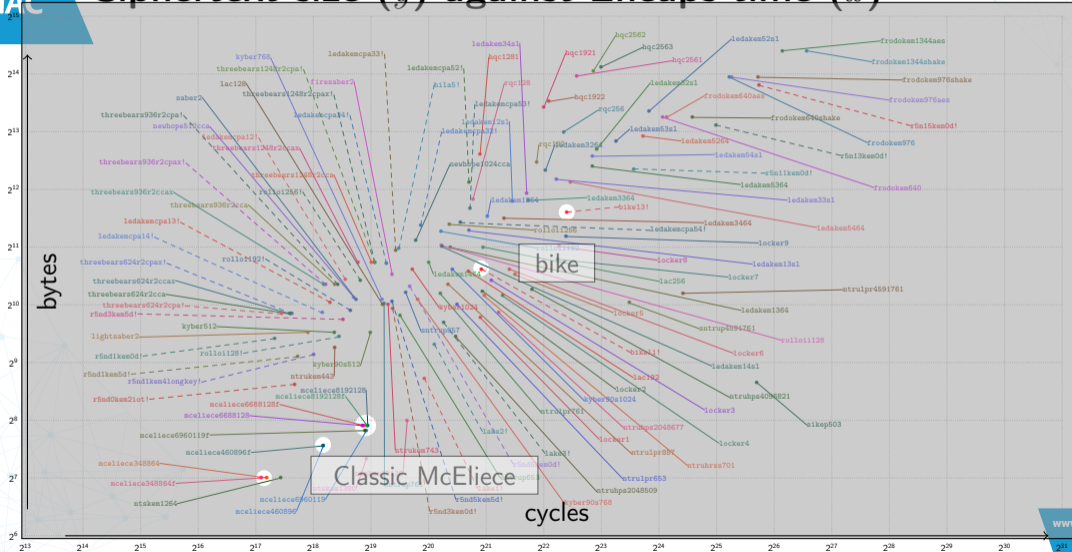
cycles

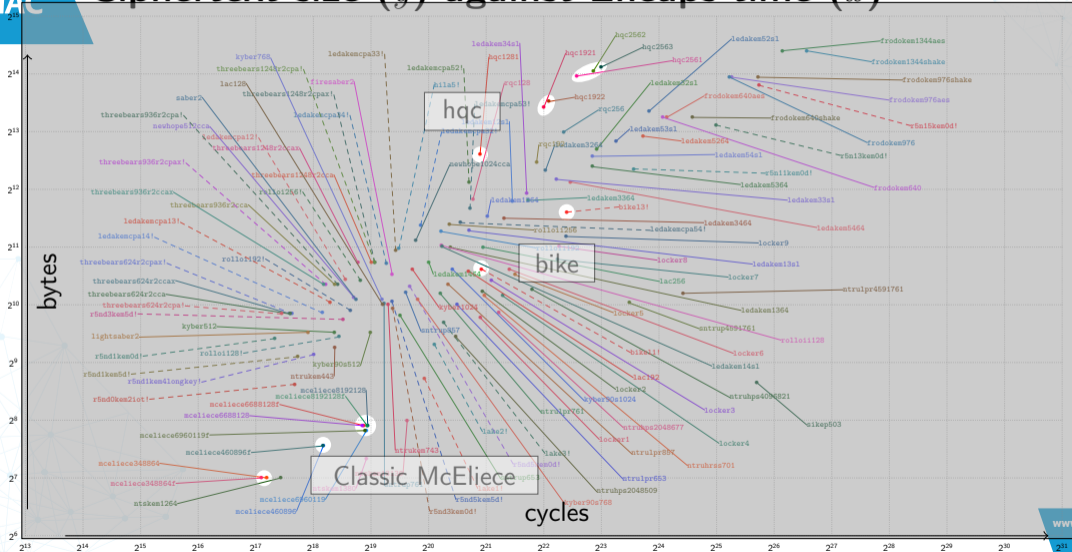
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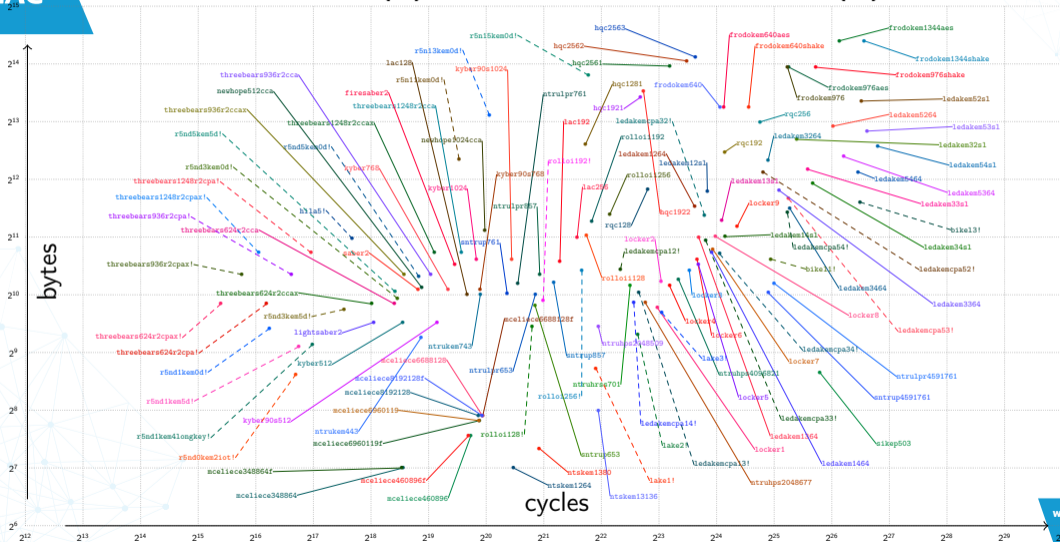
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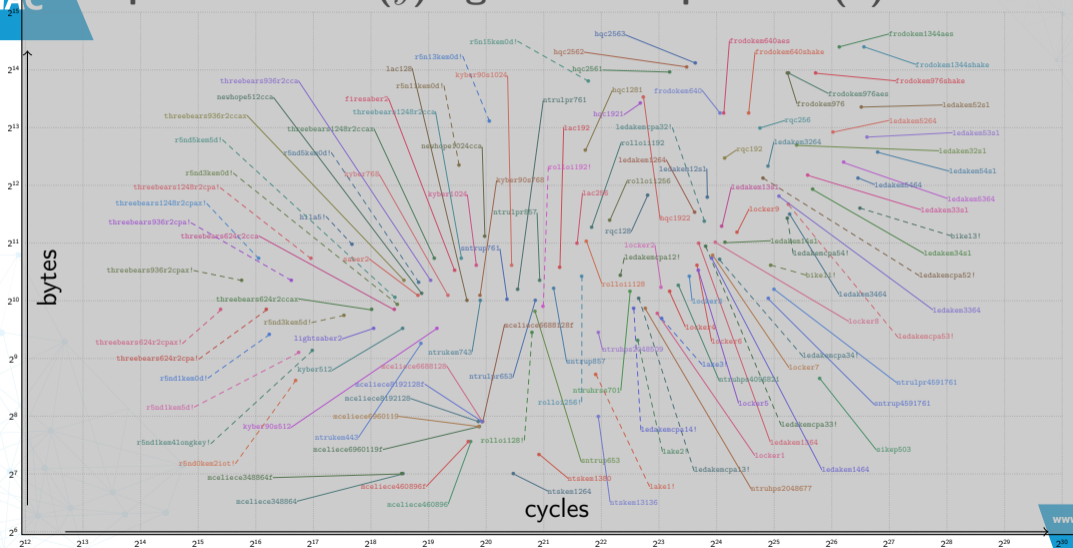
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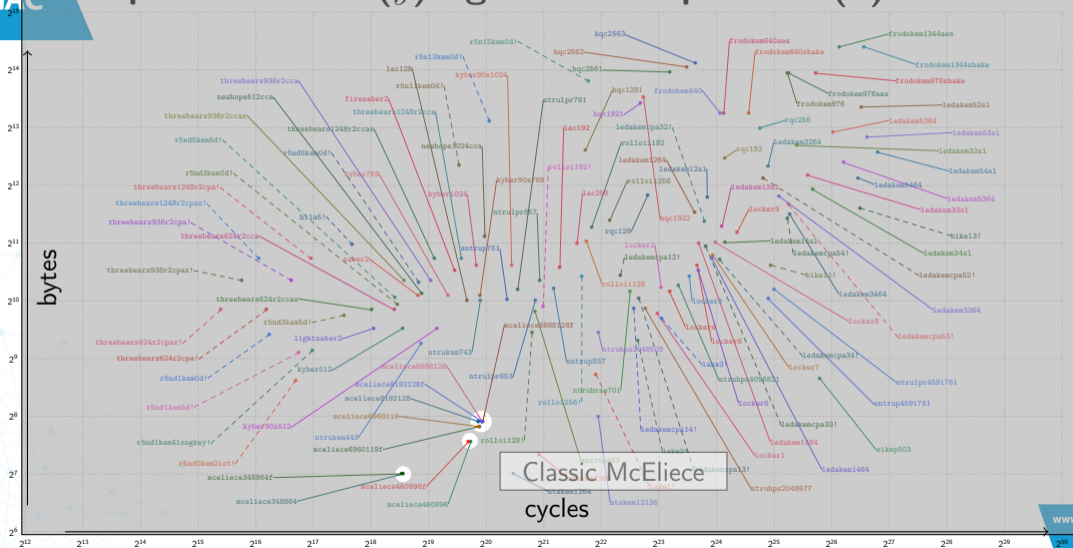
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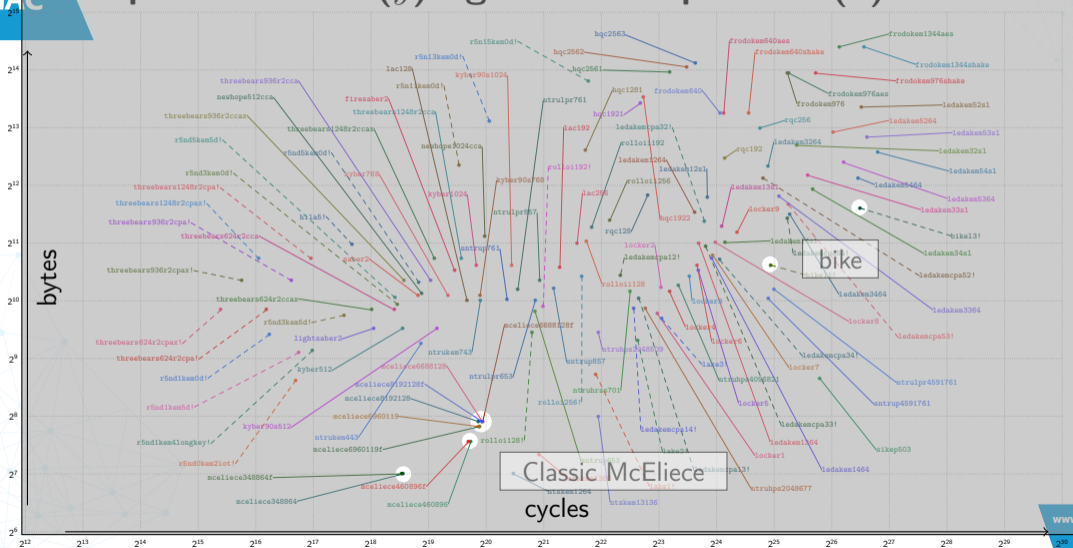
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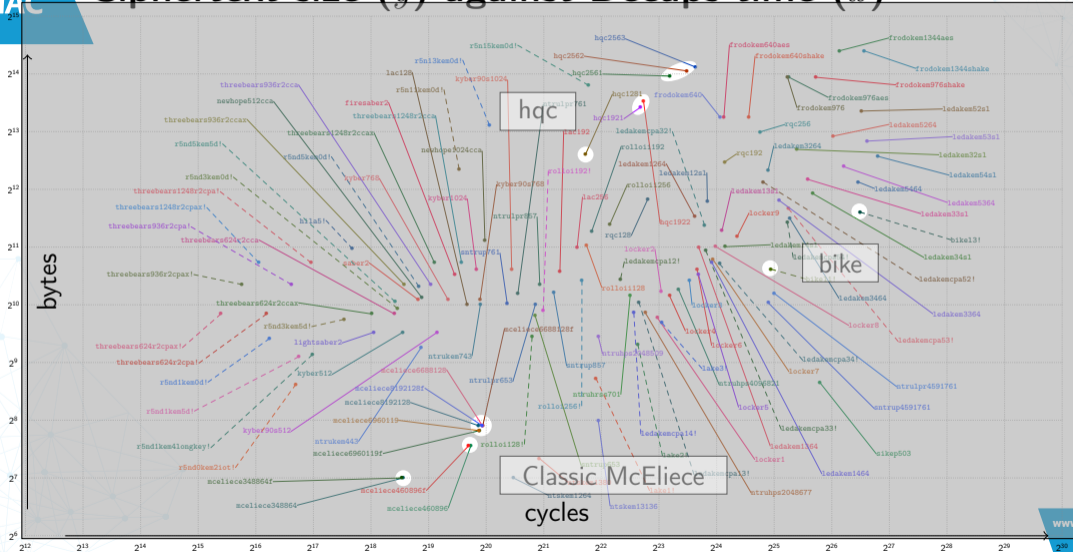




# Ciphertext size ( $y$ ) against Decaps time ( $x$ )







# Energy-consumption (mJ)

		KeyGen	Encaps	Decaps
Classic McEliece	128	6384.90	1.84	588.10
	192	11632.40	3.09	1497.50
	256	38234.60	4.81	2625.30
BIKE	128	11.85	2.70	46.30
	192	38.29	8.75	119.60
	256	85.95	21.54	270.70
HQC	128	8.76	18.42	27.03
	192	25.68	41.81	70.26
	256	49.80	87.53	145.55

# Ongoing work: hardware implementation for HQC

For security level 1, targeting 128 bits of security

		LUT	FF	Slices	BRAM	Freq	kcycles	$\mu s$
Classic McEliece	KeyGen	25 327	49 383	—	168	108	1 600	14 800
	Encaps	25 327	49 383	—	168	108	2.7	25.2
	Decaps	25 327	49 383	—	168	108	18.3	169.8
BIKE	KeyGen	29 448	5 498	8 419	28	96	259	2 691
	Encaps	29 448	5 498	8 419	28	96	12	127
	Decaps	29 448	5 498	8 419	28	96	13 120	136 443
HQC*	KeyGen	1 589	1 369	580	15	150	80	528
	Encaps	2 817	2 720	1 165	22	150	162	1 067
	Decaps	5 726	4 612	2 066	46	150	225	1 487

\* preliminary results, simulation only...

# Brief summary of the last CBC candidates' features

Classic McEliece	BIKE	HQC
Algebraic codes in H. metric	Non-algebraic codes in Hamming metric	
binary Goppa codes	Quasi-Cyclic Moderate Density Parity-Check Codes	
<ul style="list-style-type: none"> <li>■ longevity</li> <li>■ super fast encrypt</li> <li>■ ridiculously small ct</li> <li>■ fast decrypt</li> <li>■ biggest pk</li> <li>■ slowest KeyGen</li> <li>■ energy-consuming</li> </ul>	<ul style="list-style-type: none"> <li>■ originally proposed in 2012</li> <li>■ small pk</li> <li>■ reasonable ct</li> <li>■ energy-efficient</li> <li>■ slow decrypt</li> <li>■ slow KeyGen</li> </ul>	<ul style="list-style-type: none"> <li>■ reasonable pk</li> <li>■ fast KeyGen</li> <li>■ reasonable encrypt</li> <li>■ energy-efficient</li> <li>■ security assumption</li> <li>■ decryption failure analysis</li> <li>■ hardware compact</li> <li>■ somehow young (2016)</li> <li>■ pk/ct larger than BIKE</li> </ul>



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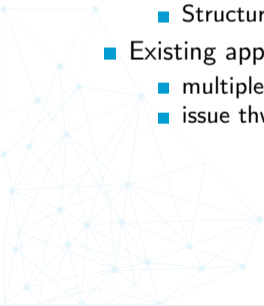
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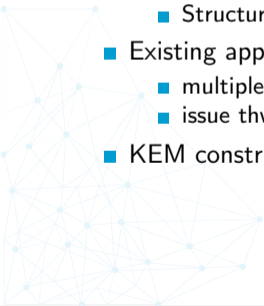




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Code-based crypto is ready, mature enough for standardization!



# Aspects that could/should be improved

Despite its maturity, CBC could benefit from:

- additional practical cryptanalysis assessment





# Aspects that could/should be improved

Challenges for code-based

Non sécurisé | decodingchallenge.org

Home Generic problems NIST-like problems Documentation Contact

## Welcome to the code-based challenges webpage!

The purpose of this webpage is to assess the **practical hardness** of problems in coding theory.

### The Challenges

The following challenges are currently available.

**Generic problems.** First we consider the two main problems on which Hamming-weight code-based cryptography mainly relies.

> **Challenge: Syndrome Decoding**

Given a matrix  $\mathbf{H} \in \mathbb{F}_2^{(n-k) \times n}$ , a vector  $\mathbf{s} \in \mathbb{F}_2^{n-k}$  and an integer  $w \leq n$ , find a vector  $\mathbf{e} \in \mathbb{F}_2^n$  of Hamming weight  $|\mathbf{e}| \leq w$  such that  $\mathbf{H}\mathbf{e}^T = \mathbf{s}^T$ . Here we will focus especially on the case with rate  $R = 0.5$  and  $w$  close to the Gilbert-Varshamov bound.

> **Challenge: Finding Low Weight Codewords**

In a random linear code of size  $n = 1280$  and rate  $R = 0.5$ , there exists on average a unique codeword of weight 144 (the Gilbert-Varshamov bound) and finding it should require at least  $2^{128}$  operations with the best known algorithms. Finding words of higher weight is easier. The goal of this challenge is to find codewords with a weight as close as possible to the GV-bound.

**NIST-like problems.** We propose challenges with the same parameter settings as the main cryptographic schemes proposed for the NIST standardization process for post-quantum cryptography. For now, we propose two such challenges in Hamming metric. In both cases, the goal is to assess the hardness of generic decoding, not to find distinguishers on the codes. Therefore we propose random linear codes with the same rate and error weight as the corresponding NIST candidates.

> **Challenge: McEliece-Goppa Syndrome Decoding**

The challenge is to solve the Syndrome Decoding problem for a random linear code of rate  $R = 0.8$  and an error-weight  $w = (1 - R)n / \log_2(n)$ . This corresponds to instances of the SD problem on which the security of the Classic McEliece cryptosystem relies.

> **Challenge: QC-MDPC Syndrome Decoding**

The challenge is to solve the Syndrome Decoding problem for a random quasi-cyclic linear code of rate  $R = 0.5$  and an error-weight  $w \approx \sqrt{n}$ . This corresponds to instances of the SD problem on which the security of the BIKE, HQC and LEDAcrypt cryptosystems rely.

**Latest news**

- 20-08-2019. Announcement of the challenge at Crypto 2019 conference
- 12-08-2019. All challenges are now online
- 21-07-2019. Website is online.
- 12-07-2019. Contact email is active.

1:

ment: see [decodingchallenge.org](https://decodingchallenge.org)



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THANKS!



# References I



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