

# Attacks on authenticated key establishment protocols with forcing the public ephemeral values

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# Security Models in Cryptography

## Cryptanalysis steps:

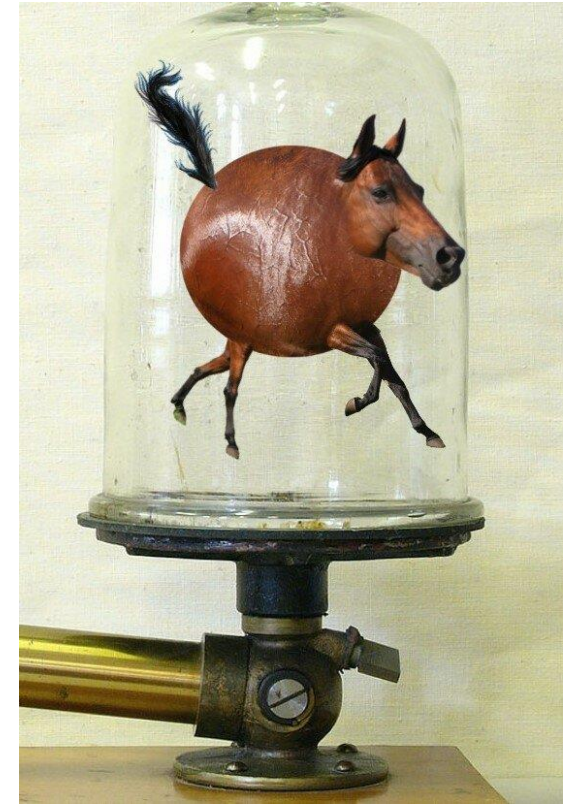
- 1) Identify a relevant security model (based on expert experience)
- 2) Describe the model formally
- 3) Get security estimation within a formal model

## Phong Q. Nguyen:

«There are a lot of similarities between cryptology and physics. Both use a lot of mathematics, but neither is part of mathematics.»

On importance of correct definition of security model:

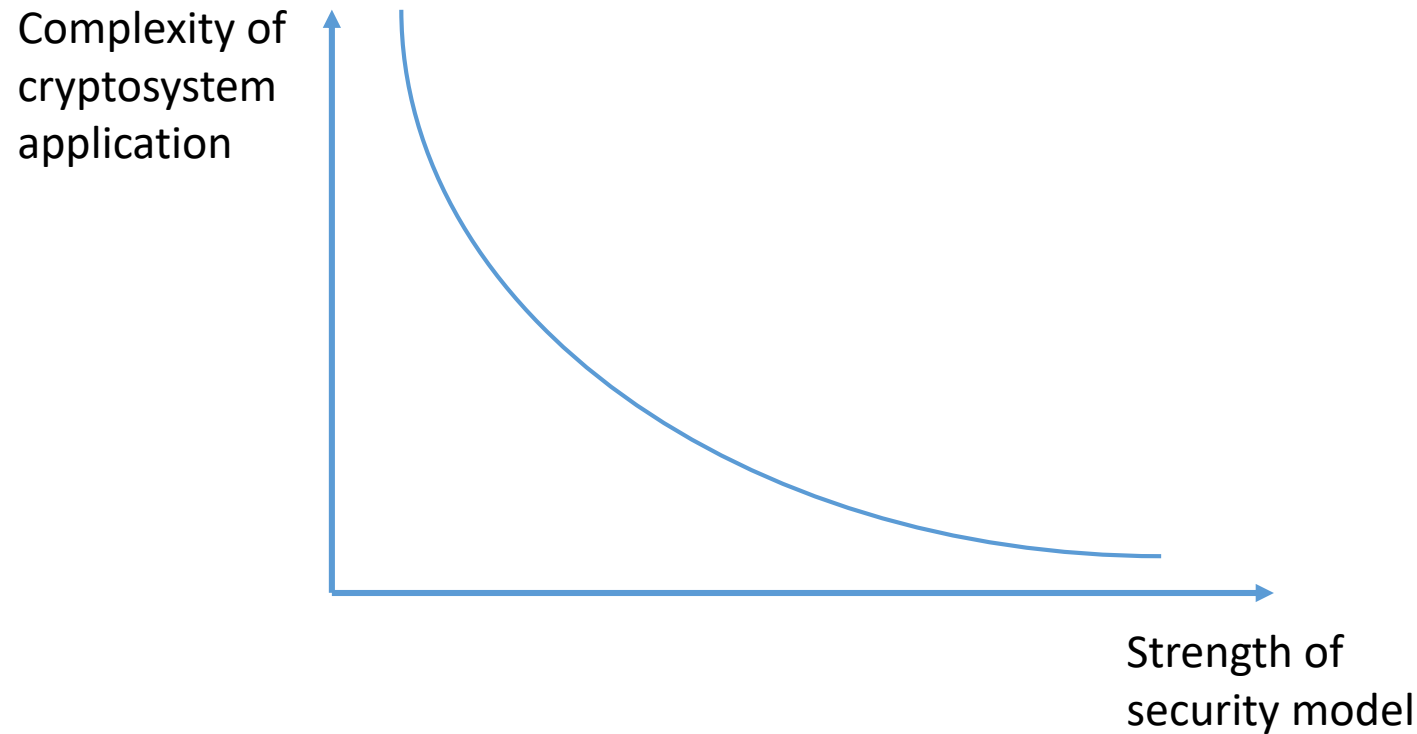
- Alekseev E.K., “What can go wrong if you use cryptography incorrectly”, CTCrypt 2019
- Degabriele J.P., Paterson K., Watson G. “Provable Security in the Real World”



# Sources of Adversary Capabilities

Source of adversary capabilities:

- how cryptosystem is used/supposed to be used in practice



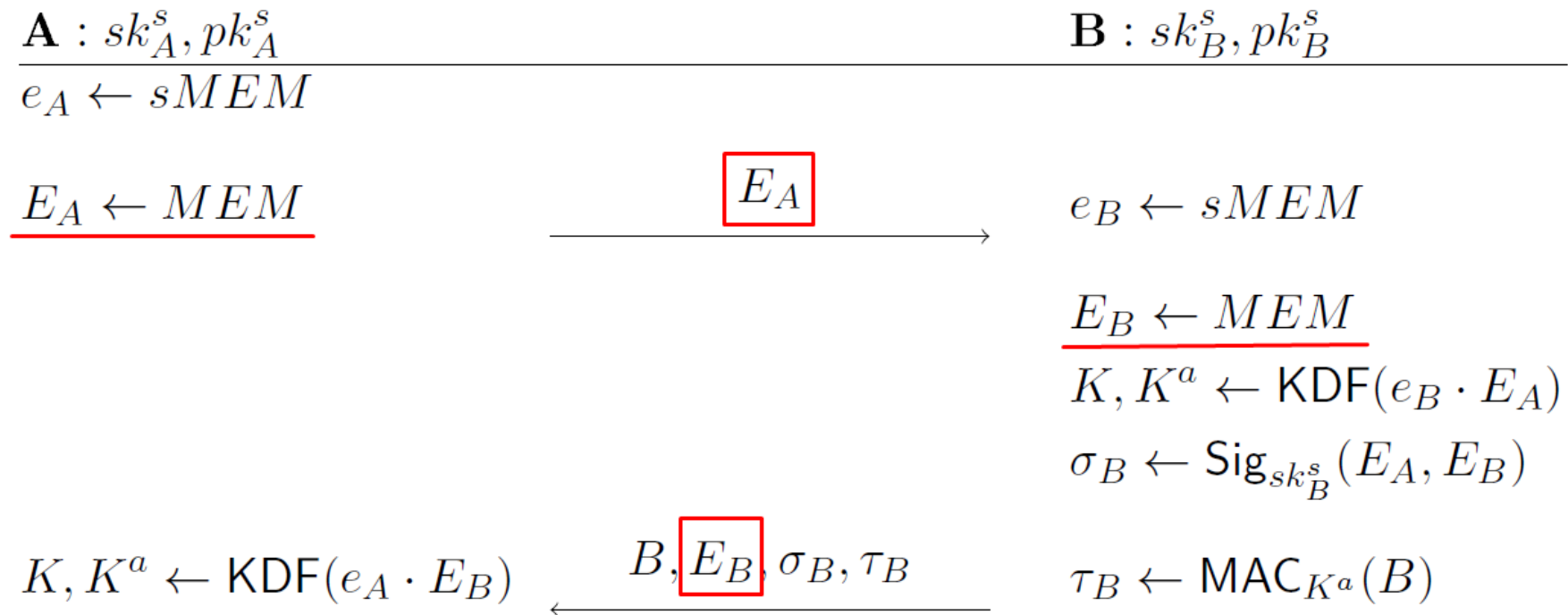
# Attack on SIGMA

Public ephemeral values:

- appears during the protocol execution
- transmitted over the channel

Examples:

- client\_random, server\_random in TLS
- ephemeral public keys in any DH-based AKE

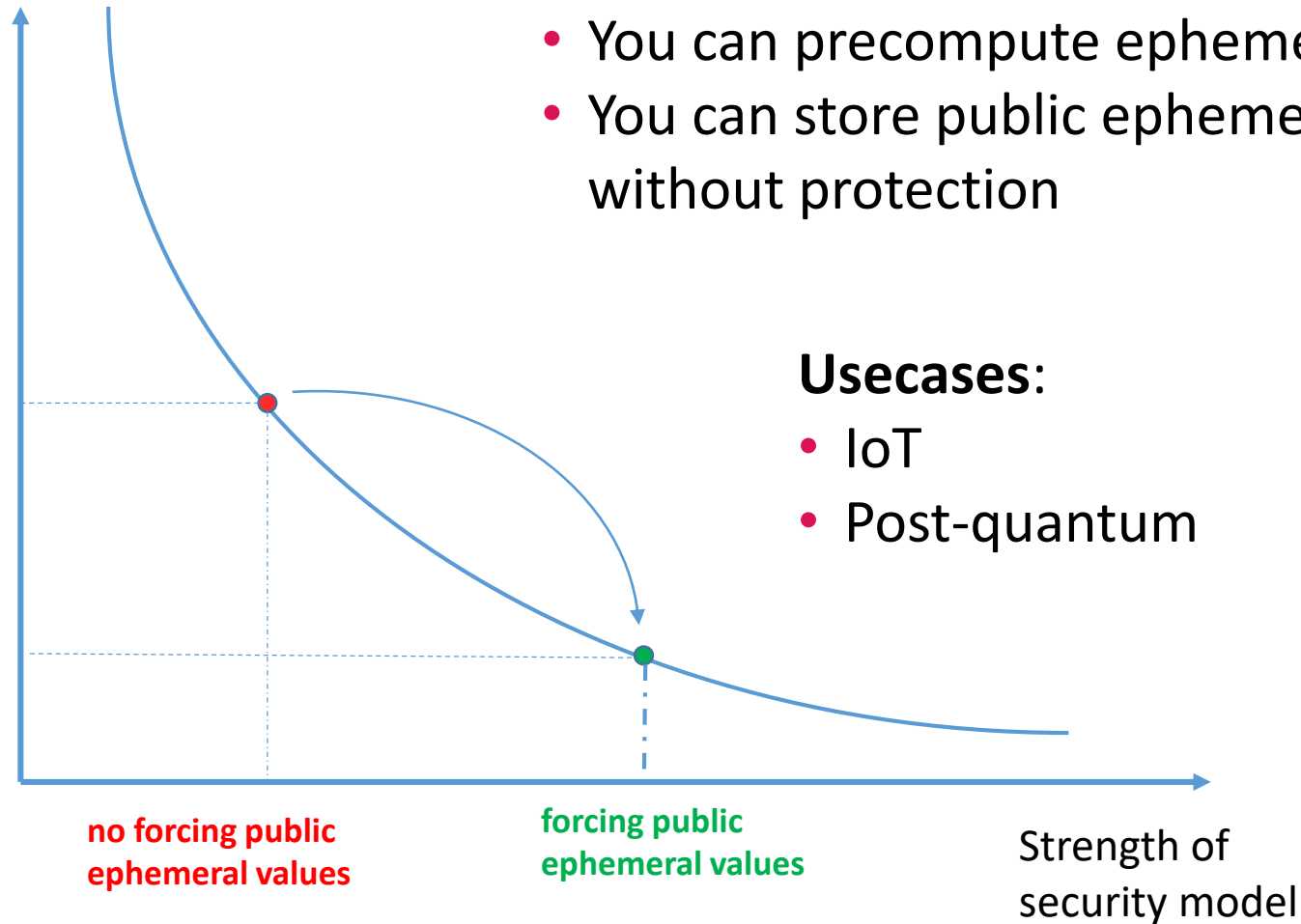


# Forcing the public ephemeral values

Complexity of  
cryptosystem  
application

no precomputation

precomputation with  
partially unprotected storing



# Attacks on AKE-protocols

Types of protocols under consideration (based on types of long-term keys used):

- Signature
- Scalar
- KEM (Key Encapsulation Mechanism)

Implemented threats:

- AUTH – adversary impersonate one of participants
- MITM – adversary impersonate both participants
- KCI – adversary impersonate some participant knowing the other participant's long-term key
- PFS – adversary gets the keys of previously established sessions after getting some participant's long-term key
- SEC – adversary gets the session key established by some two honest participants

# Attacks on AKE-protocols

Protocol type	Protocol	Result
Signature	SIG-DH+	AUTH
	SIGMA	AUTH, MITM
	Echinacea-3	AUTH, MITM
	SIGMA-R	AUTH
Scalar	TS3	AUTH
	CF	AUTH
	SK6	KCI, PFS, SEC
KEM	BKM-KK	AUTH

Notations: «AKE Zoo: 100 two-party protocols (to be continued)», ePrint paper 2023/1044.

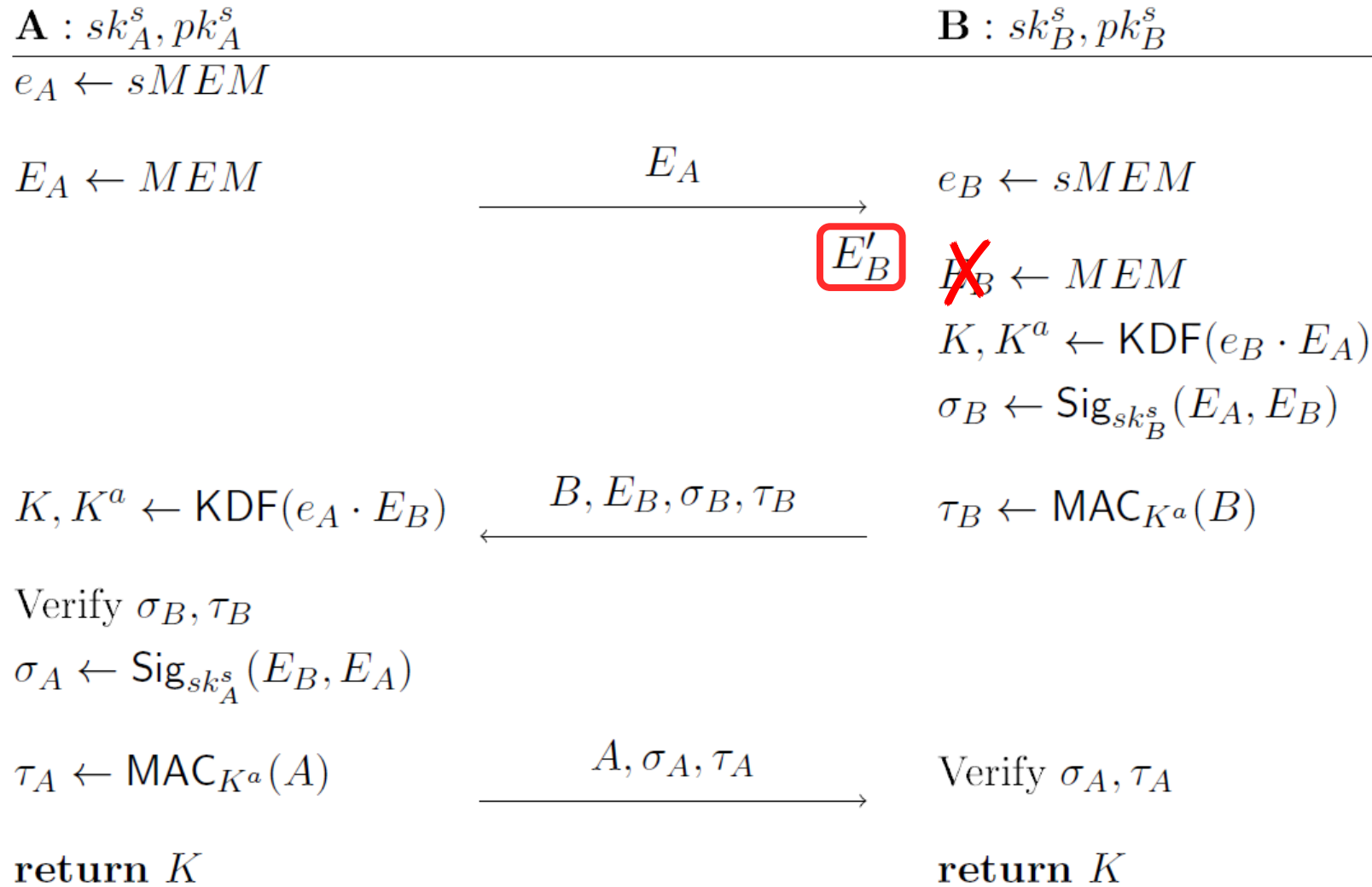
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	<b>CF</b>	<b>AUTH</b>
	SK6	KCI, PFS, SEC
KEM	<b>BKM-KK</b>	<b>AUTH</b>

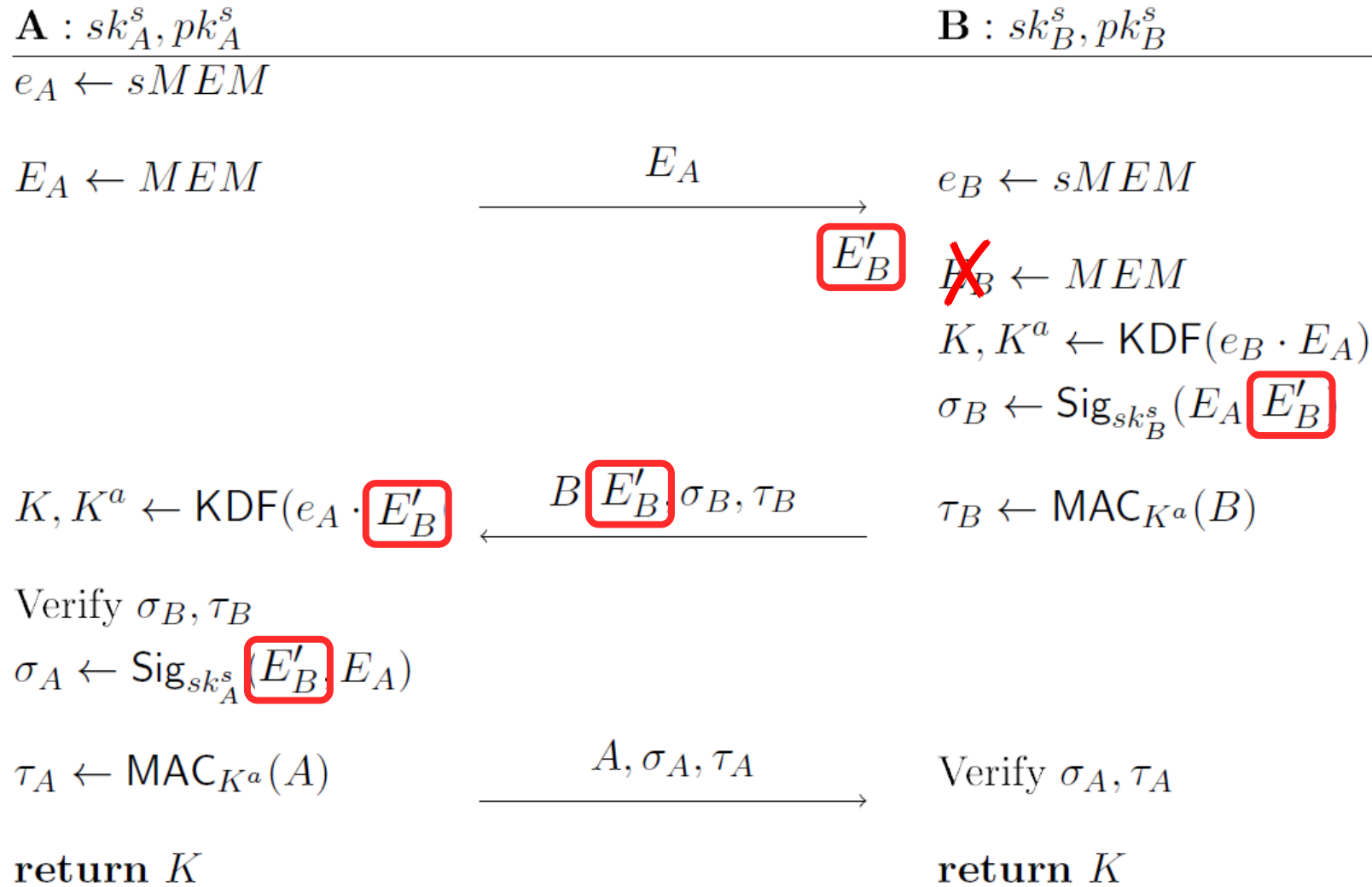
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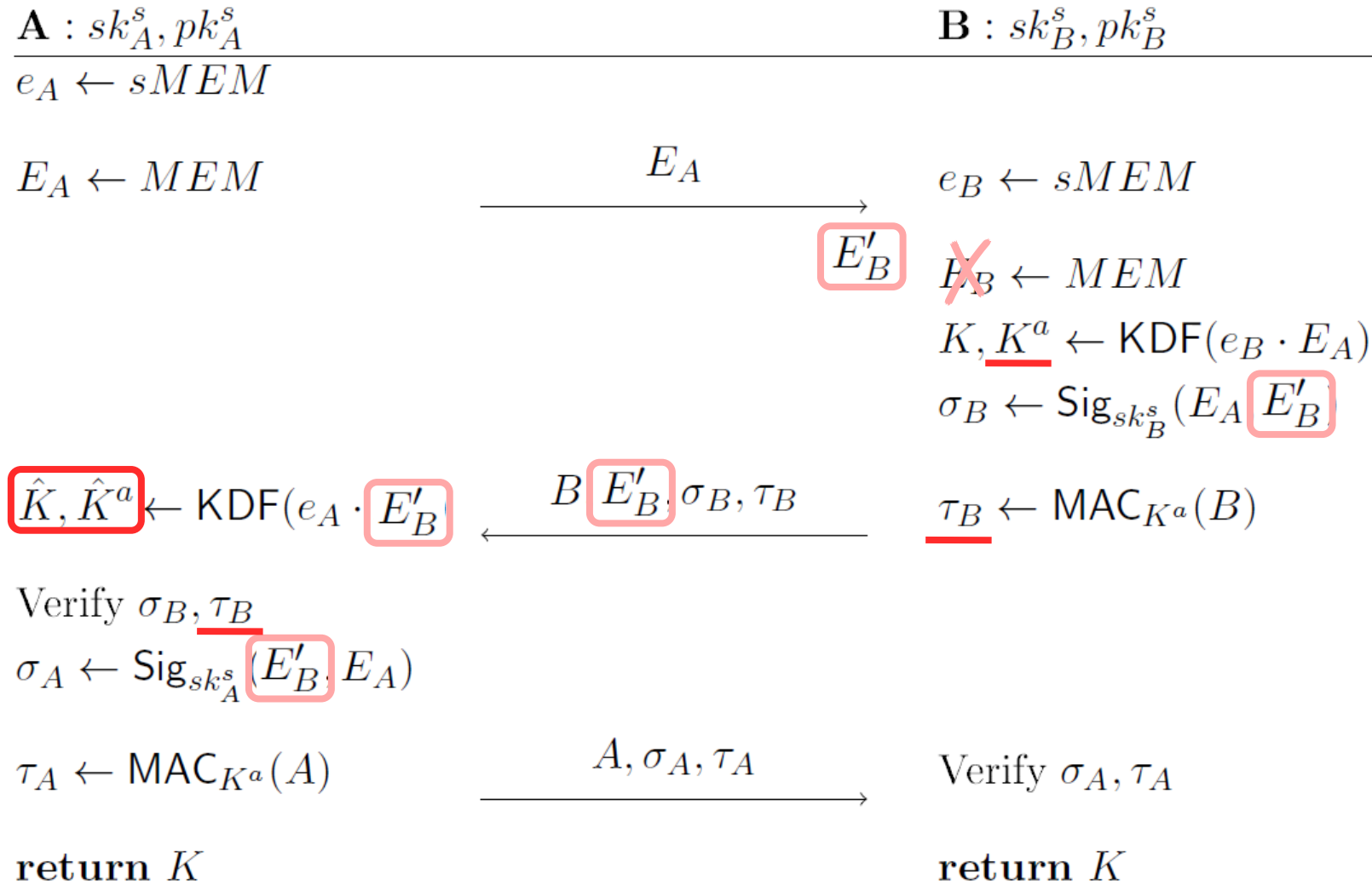
# Attack on SIGMA



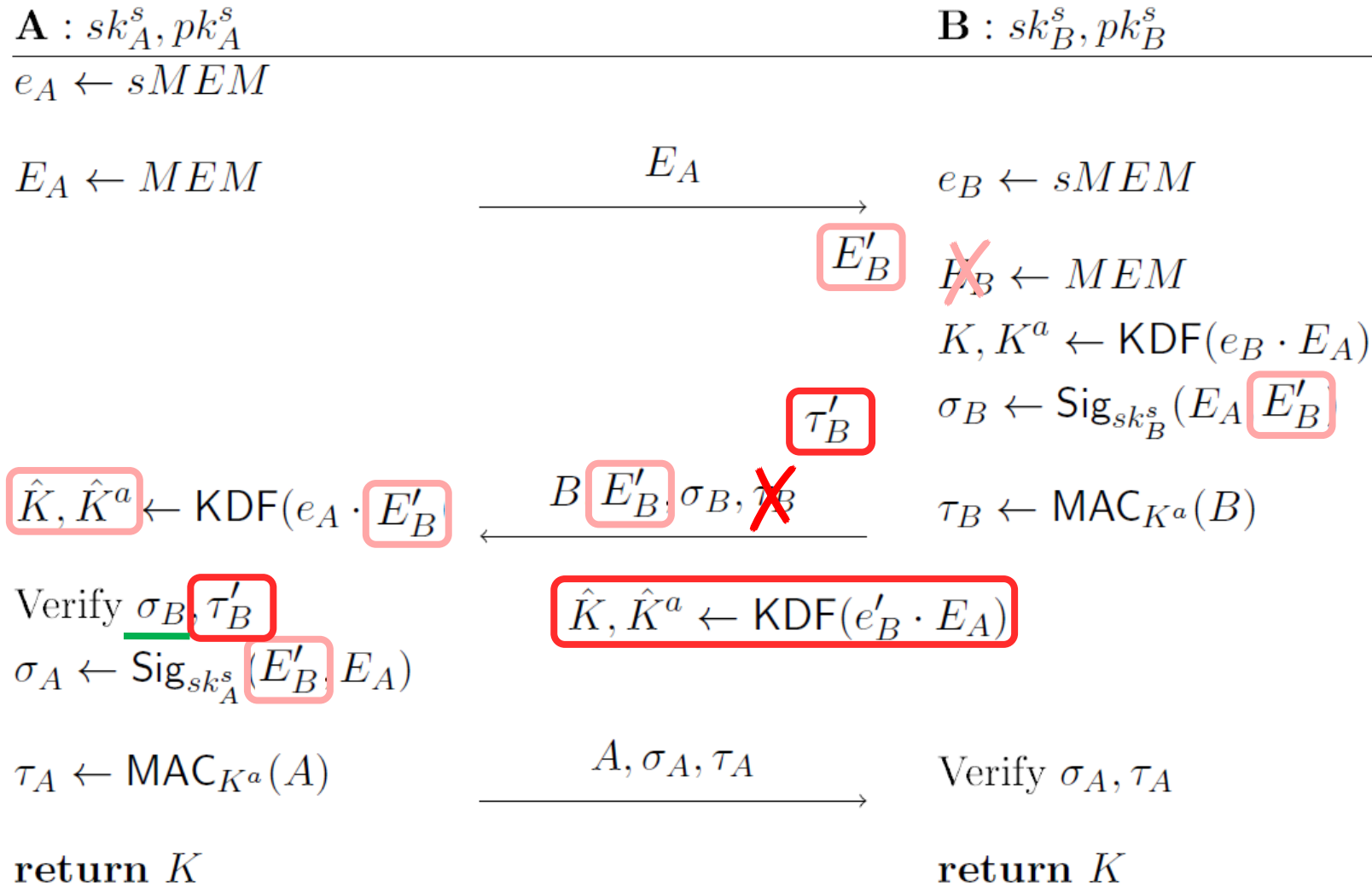
# Attack on SIGMA



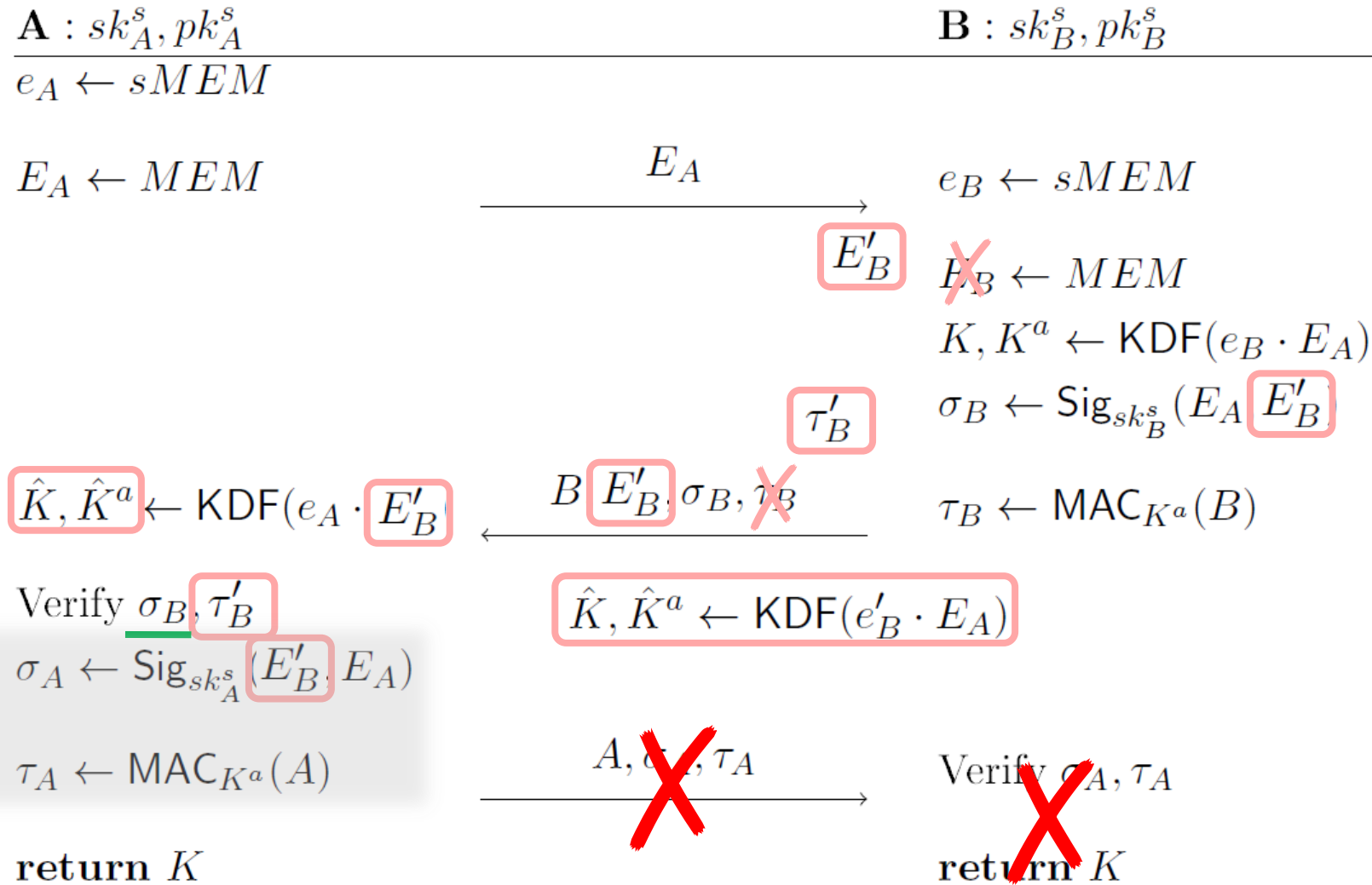
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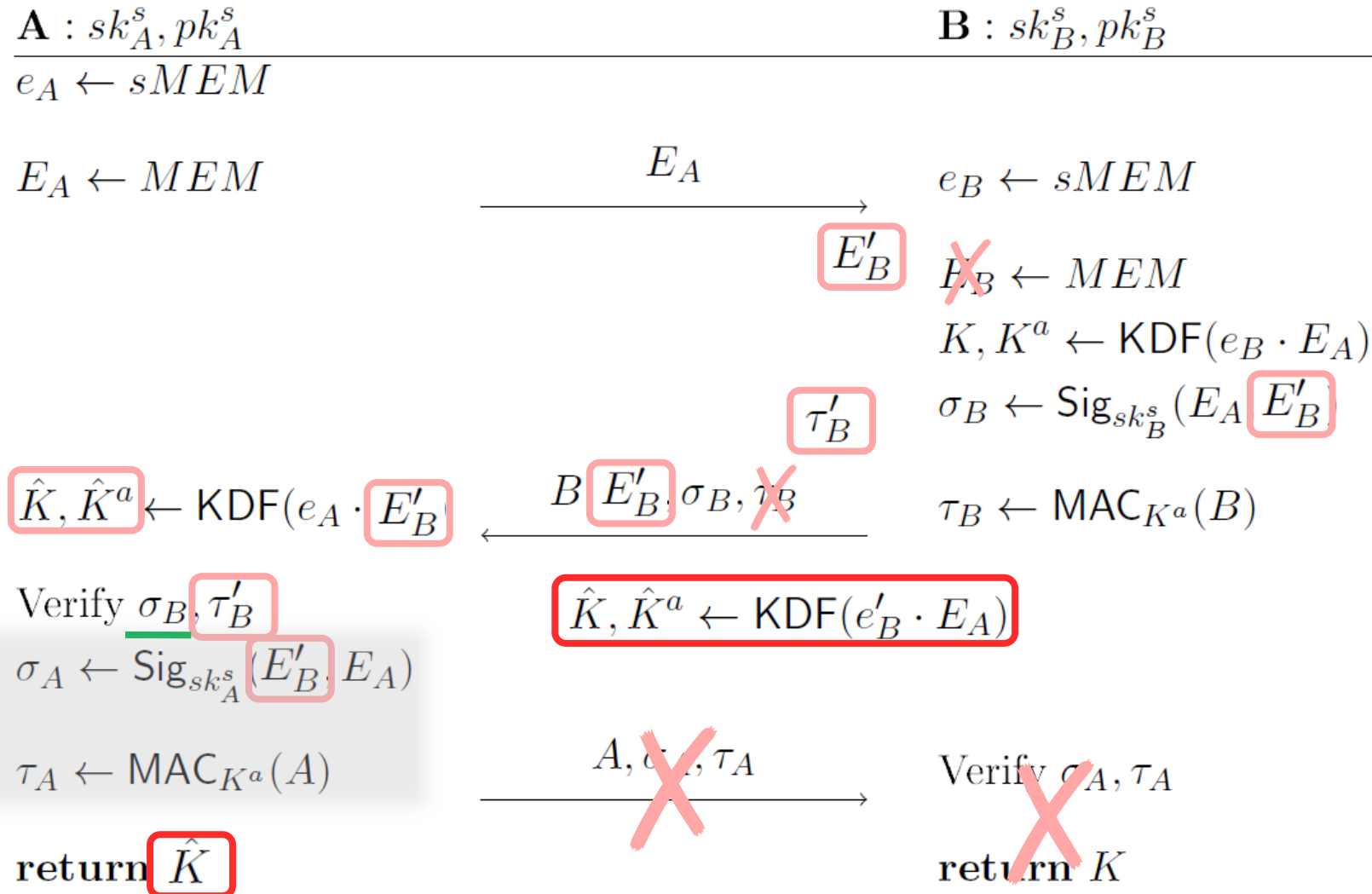
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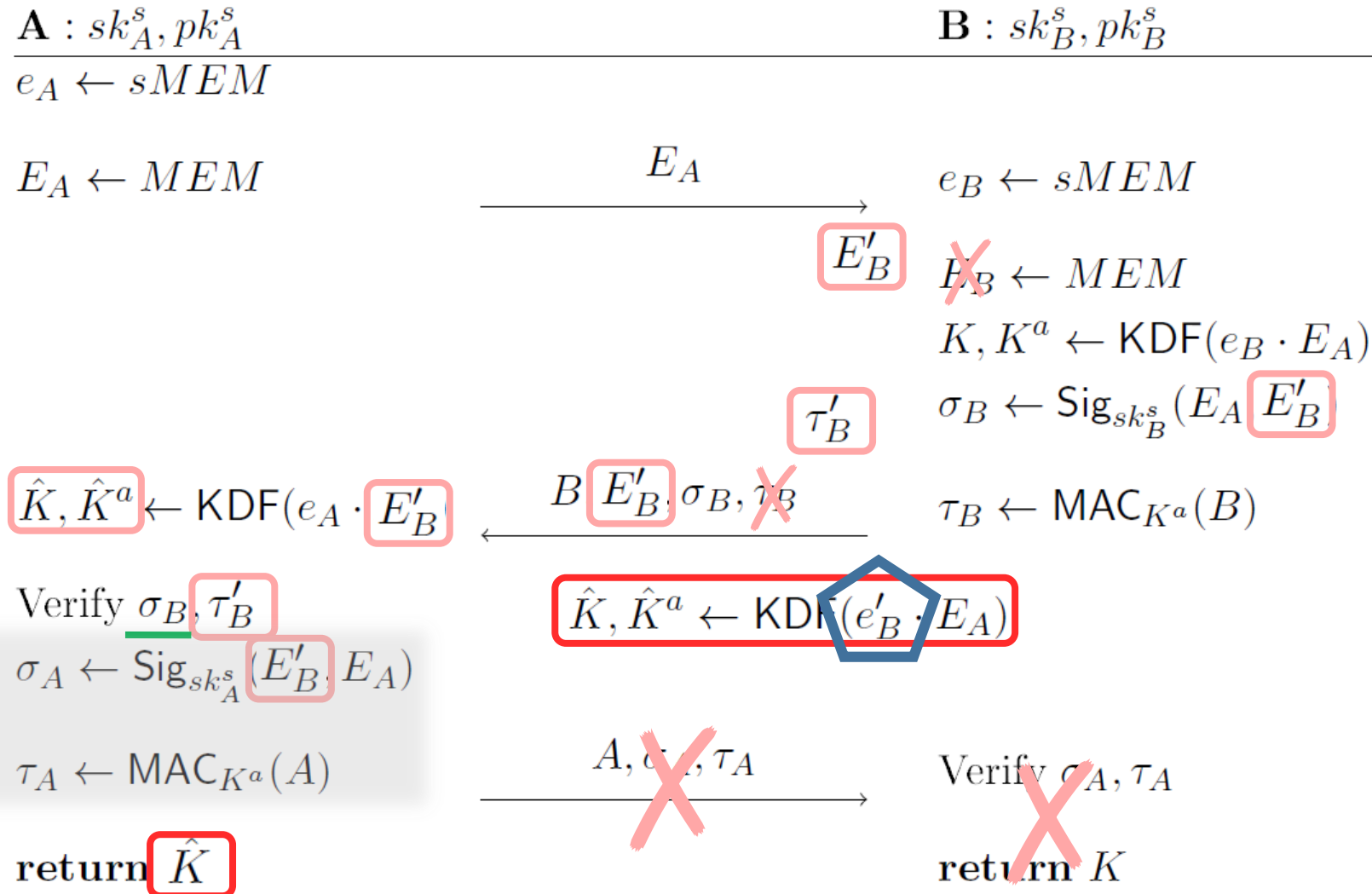
# Attack on SIGMA



# Attack on SIGMA



# Attack on SIGMA



# Attack on CF

**A** :  $(x_A, X_A)$   $(sk_A^s, pk_A^s)$

$e_A \leftarrow sMEM$

$E_A \leftarrow MEM$

$\sigma_A \leftarrow \text{Sig}_{sk_A^s}(E_A)$

$A, E_A, \sigma_A$

**B** :  $(x_B, X_B)$   $(sk_B^s, pk_B^s)$

Verify  $\sigma_A$

$e_B \leftarrow sMEM$

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$\sigma_B \leftarrow \text{Sig}_{sk_B^s}(E_B)$

Verify  $\sigma_B$

$B, E_B, \sigma_B$

$W \leftarrow (e_A + x_A) \cdot (E_B + X_B)$

$K \leftarrow \text{KDF}(A, B, W, E_A)$

return  $K$

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$$E'_A = -X_A + P$$

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$B, E_B, \sigma_B$

~~Verify  $\sigma_B$~~

~~$W \leftarrow (e_A + x_A) \cdot (E_B + X_B)$~~

~~$K \leftarrow \text{KDF}(A, B, W, E_A)$~~

~~return  $K$~~

$W' \leftarrow E_B + X_B$

$\hat{K} \leftarrow \text{KDF}(A, B, W', E'_A)$

$W \leftarrow (e_B + x_B) \cdot (E'_A + X_A)$

$K \leftarrow \text{KDF}(A, B, W, E'_A)$

return  $\hat{K}$

# Attack on BKM-KK (2023, post-quantum!)

**A** :  $sk_A^k, pk_A^k$

$C_A, K_A^a \leftarrow \mathcal{KEM}.Encaps_{pk_B^k}()$

$C_A$

Verify  $\tau_B$

$K_B^a \leftarrow \mathcal{KEM}.Decaps_{sk_A^k}(C_B)$

$C_A^*, K \leftarrow \mathcal{KEM}.Encaps_{\widetilde{pk_B^k}}()$

$\tau_A \leftarrow \text{MAC}_{K_B^a}(C_A^*, C_A, C_B, B)$

$C_A^*, \tau_A$

return  $K$

**B** :  $sk_B^k, pk_B^k$

$\widetilde{sk_B^k} \leftarrow sMEM$

$\widetilde{pk_B^k} \leftarrow MEM$

$C_B, K_B^a \leftarrow \mathcal{KEM}.Encaps_{pk_A^k}()$

$K_A^a \leftarrow \mathcal{KEM}.Decaps_{sk_B^k}(C_A)$

$\tau_B \leftarrow \text{MAC}_{K_A^a}(\widetilde{pk_B^k}, C_A, C_B, A)$

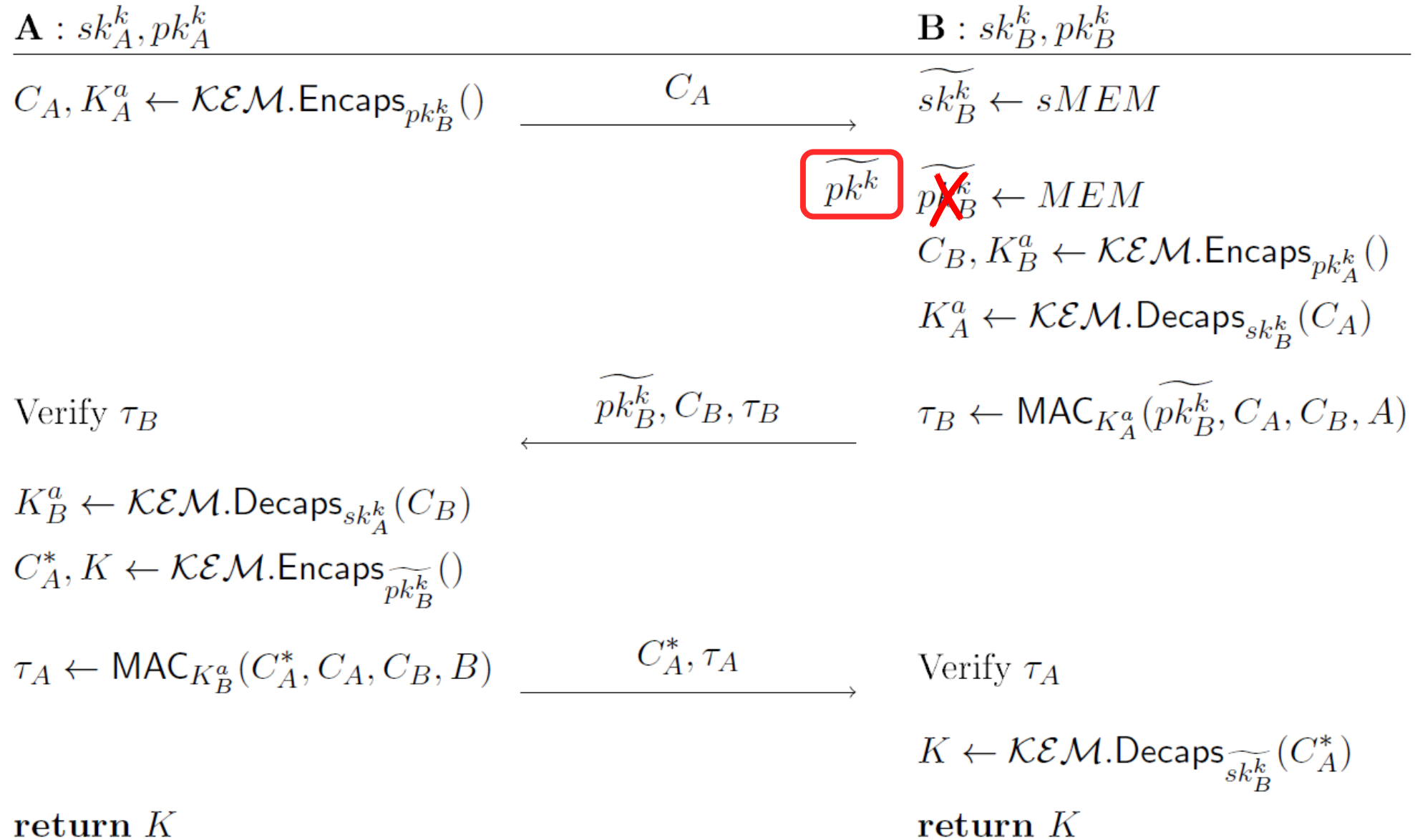
Verify  $\tau_A$

$K \leftarrow \mathcal{KEM}.Decaps_{\widetilde{sk_B^k}}(C_A^*)$

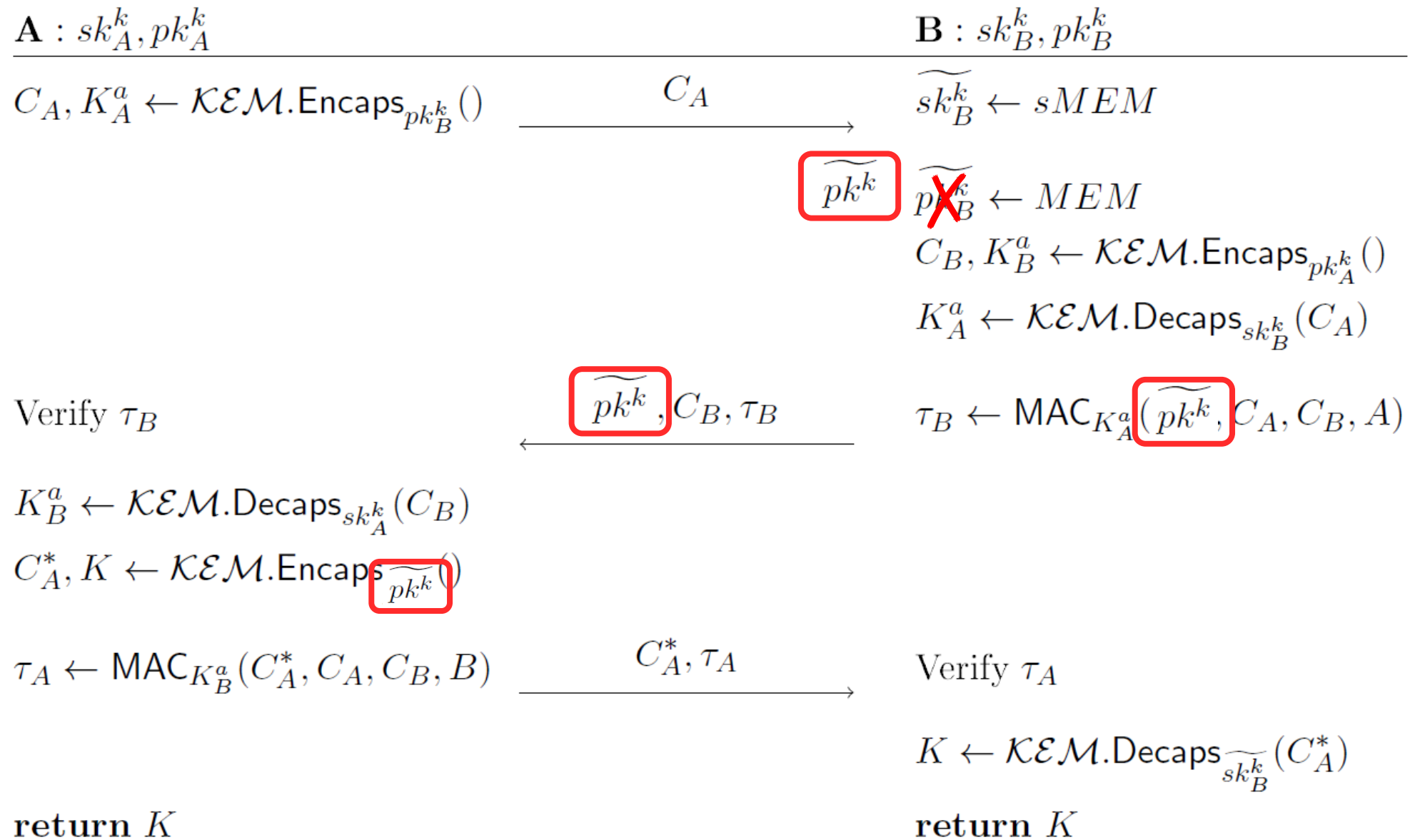
return  $K$

$\widetilde{pk_B^k}, C_B, \tau_B$

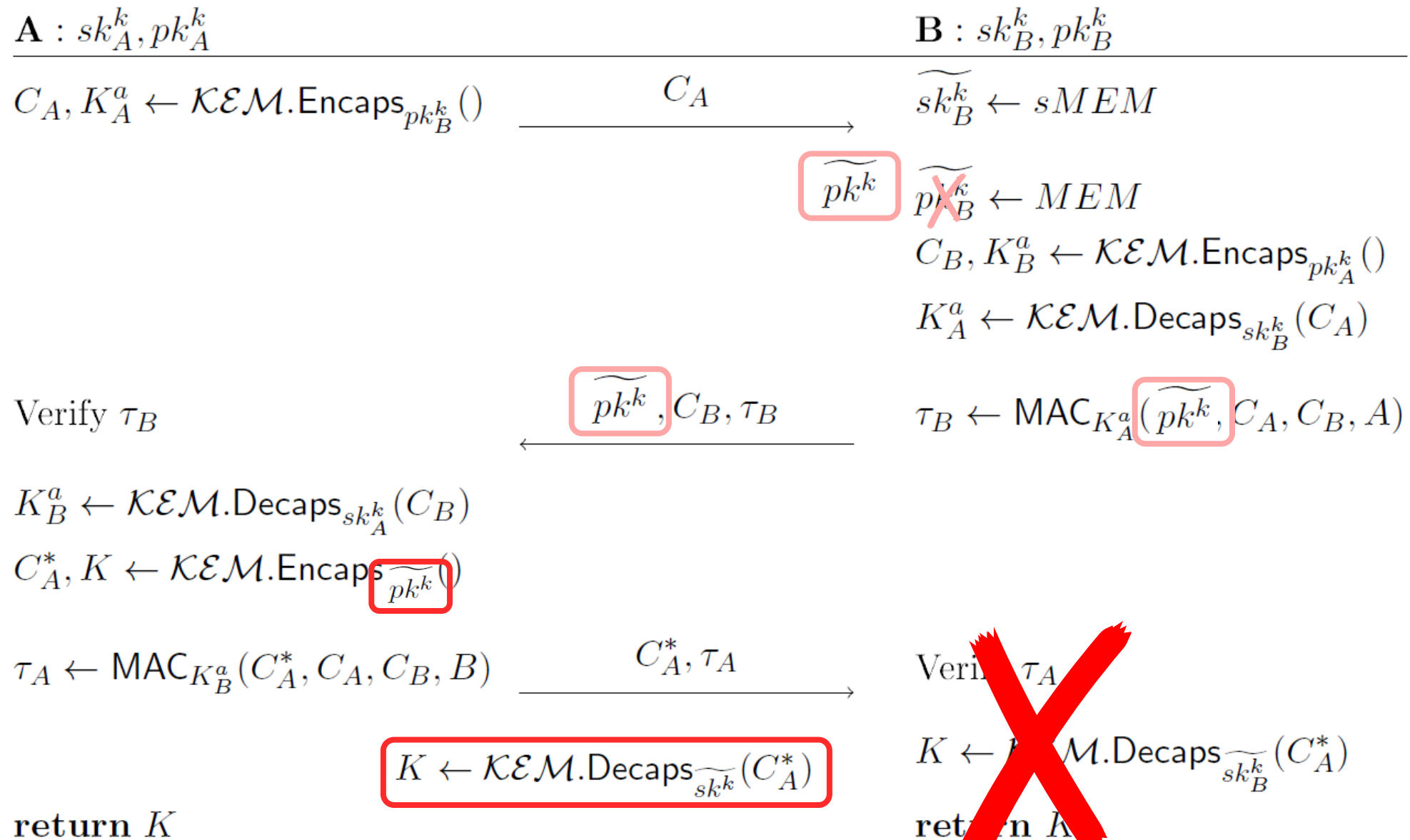
# Attack on BKM-KK (2023, post-quantum!)



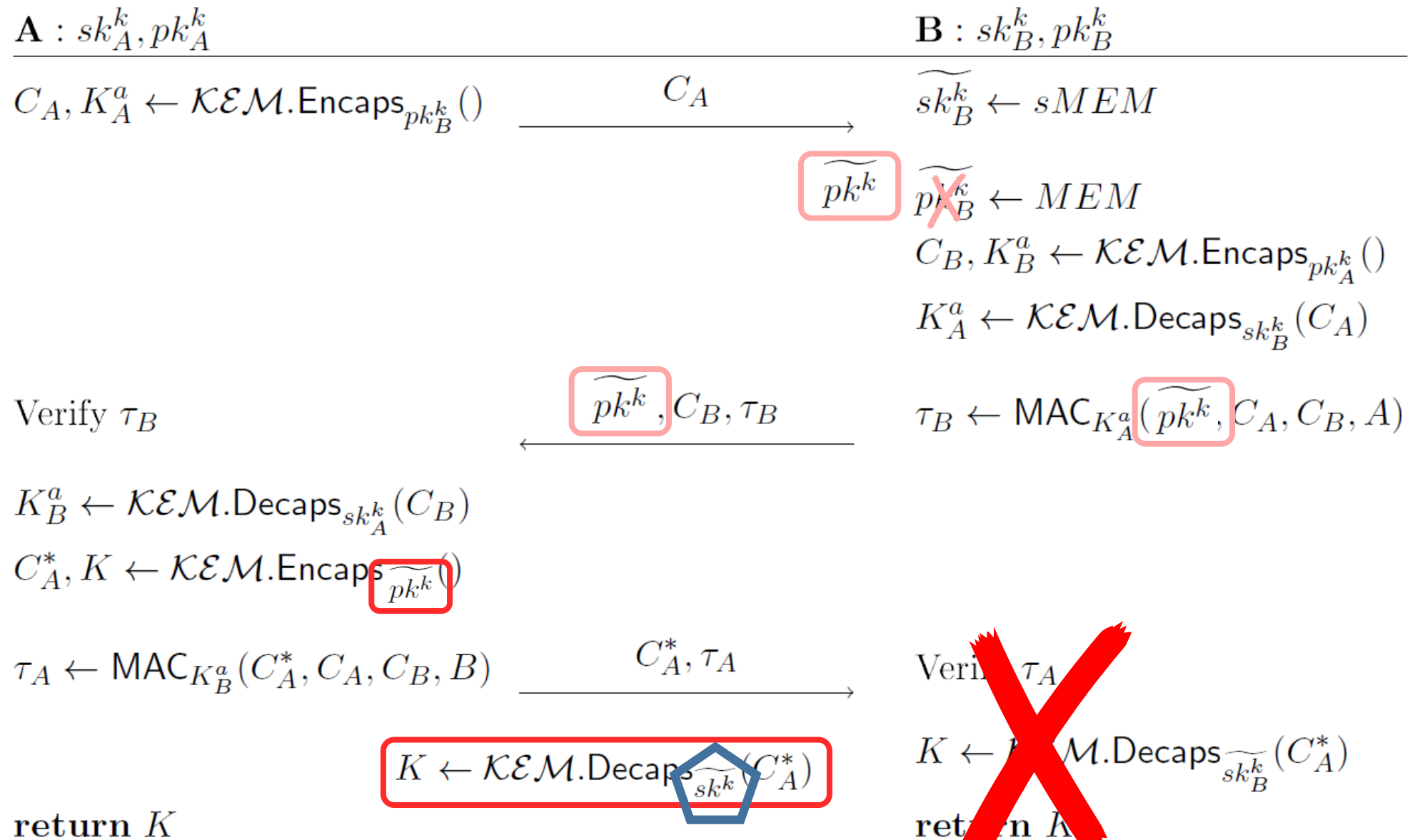
# Attack on BKM-KK (2023, post-quantum!)



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# Attack on BKM-KK (2023, post-quantum!)





# Protection for signature-based protocols

To sign values computed using ephemeral private key

$$\sigma_B \leftarrow \text{Sig}_{sk_B^s}(E_A, E_B) \quad \longrightarrow \quad \sigma_B \leftarrow \text{Sig}_{sk_B^s}(E_A, E_B, \text{MAC}_{K^a}(B))$$

**SIGMA** **SIGMA-opt1**

# Protection for scalar-based protocols

To compute key for authentication tags using combinations of long-term and ephemeral keys without simple algebraic relations

$$\begin{aligned} W &\leftarrow (e_B + x_B) \cdot (E_A + X_A) \\ K &\leftarrow \text{KDF}(A, B, W, E_A) \end{aligned}$$

**CF**



$$\begin{aligned} Q &\leftarrow e_B \cdot X_A \\ R &\leftarrow x_B \cdot E_A \\ K, K^a &\leftarrow \text{KDF}(Q, R, A, B) \end{aligned}$$

**Limonnik-3**

# Protection for KEM-based protocols

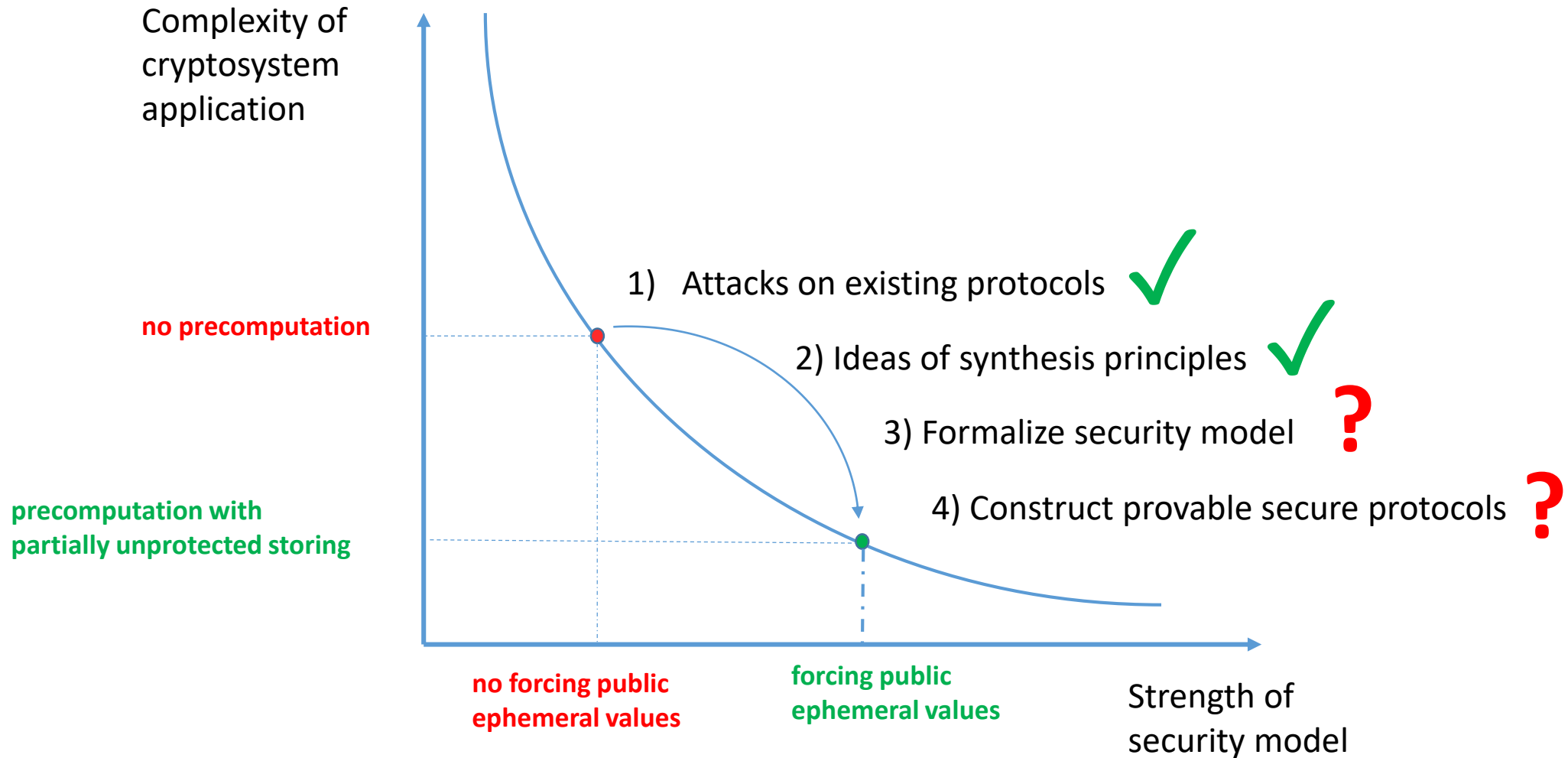
To compute key for authentication tags using results of decapsulation on long-term keys

$$\begin{array}{ccc} K \leftarrow \mathcal{KEM}.\text{Decaps}_{\widetilde{sk_B^k}}(C_A^*) & \longrightarrow & \widetilde{K_B} \leftarrow \mathcal{KEM}.\text{Decaps}_{\widetilde{sk_A^k}}(\widetilde{C_B}) \\ & & C_A, K_A \leftarrow \mathcal{KEM}.\text{Encaps}_{pk_B^k}(\cdot) \\ & & K_B \leftarrow \mathcal{KEM}.\text{Decaps}_{sk_A^k}(C_B) \\ & & K, K^a \leftarrow \text{KDF}(\widetilde{K_B}, K_A, K_B) \end{array}$$

**BKM-KK**

**KEMTLS**

# Conclusion and open questions



# Thank you for your attention!

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# Attacks on AKE-protocols

Types of protocols under consideration (based on types of long-term keys used):

- Signature
- Scalar
- KEM (Key Encapsulation Mechanism)

Used capabilities:

- FPVi/FPVr – to force ephemeral public values (to initiator/to responder)
- AC – to change the messages, transmitted over the channel
- LKC – to compromise long-term key
- SKC – to compromise session key

Implemented threats:

- AUTH – adversary impersonate one of participants
- MITM – adversary impersonate both participants
- KCI – adversary impersonate some participant knowing the other participant's long-term key
- PFS – adversary gets the keys of previously established sessions after getting some participant's long-term key
- SEC – adversary gets the session key established by some two honest participants